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> restart;
> #preferences single household;
> x0:=(c)*(g+eta)^(1/2);

$$x0 := c \sqrt{g + \eta} \quad (1)$$

> #substitute budget constraint
x1:=subs(g=y-c,x0);

$$x1 := c \sqrt{y - c + \eta} \quad (2)$$

> #optimal food consumption;
x2:=solve(diff(x1,c),c);


$$x2 := \frac{2}{3} y + \frac{2}{3} \eta \quad (3)$$

> #foodshare is decreasing in income
x2a:=x2/y;
simplify(diff(x2a,y));

$$\begin{aligned} x2a &:= \frac{\frac{2}{3} y + \frac{2}{3} \eta}{y} \\ &\quad - \frac{2}{3} \frac{\eta}{y^2} \end{aligned} \quad (4)$$

> #joint household--maximize sum of utilities but combine income
and share public good ;
#delta is cost of joint residence;
x3:=(c1)*(2*y-c1-c2+eta)^(1/2)+(c2)*(2*y-c1-c2+eta)^(1/2)-delta;

$$x3 := c1 \sqrt{2y - c1 - c2 + \eta} + c2 \sqrt{2y - c1 - c2 + \eta} - \delta \quad (5)$$

> #focus on total food consumption
> x4:=simplify(subs(c1=c-c2,x3));

$$x4 := \sqrt{2y - c + \eta} c - \delta \quad (6)$$

> #optimal food consumption
x5:=solve(diff(x4,c),c);


$$x5 := \frac{4}{3} y + \frac{2}{3} \eta \quad (7)$$

> #food share in joint household
> x5a:=x5/(2*y);
simplify(diff(x5a,y));

$$\begin{aligned} x5a &:= \frac{1}{2} \frac{\frac{4}{3} y + \frac{2}{3} \eta}{y} \\ &\quad - \frac{1}{3} \frac{\eta}{y^2} \end{aligned} \quad (8)$$

> #total utility in joint household;
x6:=simplify(subs(c=x5,x4));


$$x6 := \frac{4}{9} \sqrt{6y + 3\eta} y + \frac{2}{9} \sqrt{6y + 3\eta} \eta - \delta \quad (9)$$

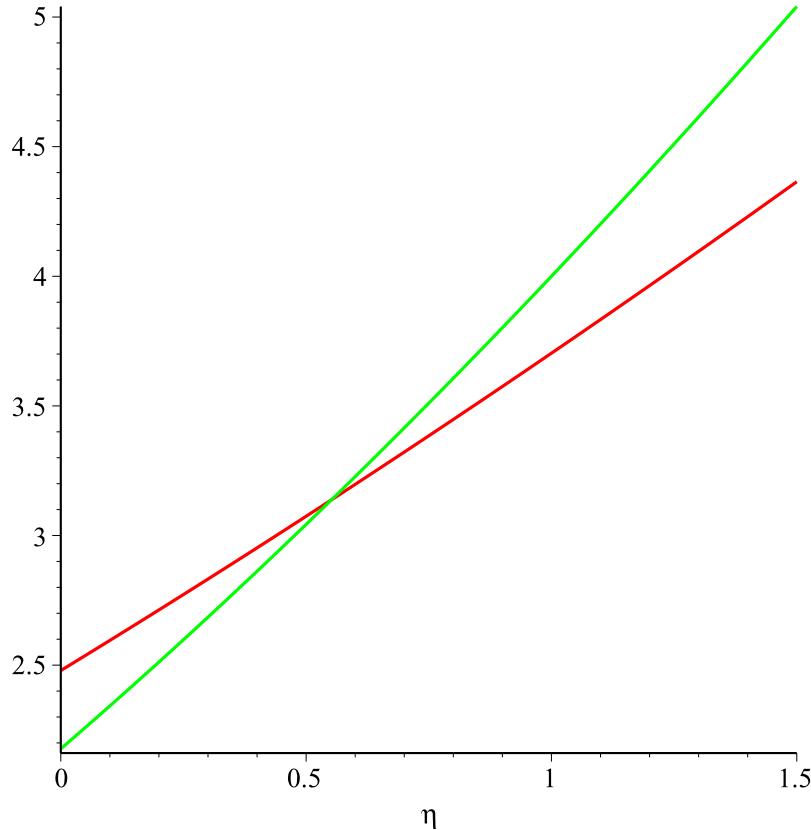

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> #combined utility if in separate households;
x7:=2*subs(c=x2,x1);

$$x7 := 2 \left( \frac{2}{3} y + \frac{2}{3} \eta \right) \sqrt{\frac{1}{3} y + \frac{1}{3} \eta} \quad (10)$$

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> #joint utility is higher when eta is higher;
plot(subs(y=2,delta=.6,[x6,x7]),eta=0..1.5,color=[red,green]);
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> x8:=solve(subs(y=2,delta=.6,x6=x7),eta);
x8 := 0.5516268246 \quad (11)
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> #foodshare is always higher in split households for given eta.
#but note that joint households only if eta>.6
#but split households only if eta<.6
#average foodshare for split households may be lower than joint.
plot(subs(y=2,[x2a,x5a,piecewise(eta<x8,x2a,x5a)]),eta=0.1..1.5,
color=[red,green,blue]);
```

