```
> restart;
```

```
> #two types of men and two types of women. A woman matched to
  third "virtual" type of man is by convention single and gets 0
  utility. Similarly for men.
```

```
> x0a:=piecewise(a<=hm[c] and b<=hf[c],a*b,0);
  x0b:=piecewise(a<=hm[c] and b<=hf[c],a*b,0);
```

$$x0a := \begin{cases} a\,b & a \le hm_c \textbf{ and } b \le hf_c \\ 0 & \mathit{otherwise} \end{cases}$$

$$x0b := \begin{cases} a\,b & a \le hm_c \textbf{ and } b \le hf_c \\ 0 & \mathit{otherwise} \end{cases} \tag{1}$$

```
> with(Optimization);
```

$$[\mathit{ImportMPS}, \mathit{Interactive}, \mathit{LPSolve}, \mathit{LSSolve}, \mathit{Maximize}, \mathit{Minimize}, \mathit{NLPSolve}, \mathit{QPSolve}] \tag{2}$$

```
> c:=2;
  c1:=c+1;
```

$$c := 2$$

$$c1 := 3 \tag{3}$$

```
> #marital untility for men and women;
  vm:=unapply(x0a,a,b);
  vf:=unapply(x0b,a,b);
```

$$vm := (a, b) \rightarrow \mathit{piecewise}\left(a \le hm_2 \textbf{ and } b \le hf_2, a\,b, 0\right)$$

$$vf := (a, b) \rightarrow \mathit{piecewise}\left(a \le hm_2 \textbf{ and } b \le hf_2, a\,b, 0\right) \tag{4}$$

```
> #levels of human capital for three types (recall third type is
  virtual).
  hm:=[1,2,4];
  hf:=[1,2,4];

  #supply of men and women by human capitaly type
  m:=[22,19];
  f:=[20,20];
```

$$hm := [1, 2, 4]$$

$$hf := [1, 2, 4]$$

$$m := [22, 19]$$

$$f := [20, 20] \tag{5}$$

```
>
```

```
>
```

```
> #total marital surplus
  x1:=sum(sum((vm(hm[i],hf[j])+vf(hm[i],hf[j]))*n[i,j],i=1..c1),j=
  1..c1);
```

$$x1 := 2\,n_{1,\,1} + 4\,n_{2,\,1} + 4\,n_{1,\,2} + 8\,n_{2,\,2} \tag{6}$$

```
> #supply and demand must equate for each type;
  x2:={'sum(n[i,j],j=1..c1)=m[i]'$'i'=1..c} ;
  x3:={'sum(n[i,j],i=1..c1)=f[j]'$'j'=1..c};
```

$$x2 := \{n_{1,1}+n_{1,2}+n_{1,3}=22,\ n_{2,1}+n_{2,2}+n_{2,3}=19\}$$

$$x3 := \{n_{1,1}+n_{2,1}+n_{3,1}=20,\ n_{1,2}+n_{2,2}+n_{3,2}=20\} \quad (7)$$

```
> #integer linear program;
  x4:=LPSolve(x1,{op(x2),op(x3)},assume={integer,nonnegative},
  maximize=true);
```

$$x4 := [196, [n_{1,1}=20, n_{1,2}=1, n_{1,3}=1, n_{2,1}=0, n_{2,2}=19, n_{2,3}=0, n_{3,1}=0, n_{3,2}=0]] \quad (8)$$

```
> #this describes marriages
  x5:=op(op(2,x4));
```

$$x5 := n_{1,1}=20, n_{1,2}=1, n_{1,3}=1, n_{2,1}=0, n_{2,2}=19, n_{2,3}=0, n_{3,1}=0, n_{3,2}=0 \quad (9)$$

```
> #whenever a given type of man is in two different states that
  type must be indifferent between those states
  x6:=eval(subs(x5,{'''signum(n[i,j]*n[i,j2])*(vm(hm[i],hf[j])+d[i,
  j]-vm(hm[i],hf[j2])-d[i,j2])'$'j'=j2..c1'$'j2'=1..c1'$'i'=1..c}))
  ;
```

$$x6 := \{0, -2+d_{1,3}-d_{1,2}, -1+d_{1,3}-d_{1,1}, 1+d_{1,2}-d_{1,1}\} \quad (10)$$

```
> #similarly for women
  x7:=eval(subs(x5,{'''signum(n[j,i]*n[j2,i])*(vf(hm[j],hf[i])-d[j,
  i]-vf(hm[j2],hf[i])+d[j2,i])'$'j'=j2..c1'$'j2'=1..c1'$'i'=1..c}))
  ;
```

$$x7 := \{0, 2-d_{2,2}+d_{1,2}\} \quad (11)$$

```
> #put together all the arbitrage conditions
  x8:=x6 union x7 minus {0} union {'d[c1,i]=0'$'i'=1..c} union {'d
  [i,c1]=0'$'i'=1..c};
```

$$x8 := \{-2+d_{1,3}-d_{1,2}, -1+d_{1,3}-d_{1,1}, 1+d_{1,2}-d_{1,1}, 2-d_{2,2}+d_{1,2}, d_{1,3}=0, d_{2,3}=0,$$
$$d_{3,1}=0, d_{3,2}=0\} \quad (12)$$

```
> #this gives us dowry levels. now compute mean.
  x9:=solve(x8);

  x10:=subs(x9,x5,sum(sum(n[i,j]*d[i,j],i=1..c),j=1..c)/sum(sum(n
  [i,j],i=1..c),j=1..c));
```

$$x9 := \{d_{1,1}=-1, d_{1,2}=-2, d_{1,3}=0, d_{2,2}=0, d_{2,3}=0, d_{3,1}=0, d_{3,2}=0\}$$

$$x10 := -\frac{11}{20} \quad (13)$$

```
> x11:=[''d[i,j]=0'$i=1..c'$j=1..c];
```

$$x11 := [d_{1,1}=0, d_{2,1}=0, d_{1,2}=0, d_{2,2}=0] \quad (14)$$

```
> #compute dowry as function of female type
  x12a:=subs(x11,x5,subs(x9,[[hf[i],sum(n[i,j]*d[i,j],j=1..c)/sum(n
  [i,j],j=1..c)]$i=1..c]));
```

$$x12a := \left[\left[1, -\frac{22}{21}\right], [2, 0]\right] \quad (15)$$

$$x9 := \{d_{1,1}=-1, d_{1,2}=-2, d_{1,3}=0, d_{2,2}=0, d_{2,3}=0, d_{3,1}=0, d_{3,2}=0\}$$

$$x10 := -\frac{11}{20} \tag{16}$$

```
> #now increase the human capital of men by 1
hm:=[1,3,4];
hf:=[1,2,4];
```

$$hm := [1, 3, 4]$$
$$hf := [1, 2, 4] \tag{17}$$

```
> x1:=sum(sum((vm(hm[i],hf[j])+vf(hm[i],hf[j]))*n[i,j],i=j..c1),j=
1..c);
```

$$x1 := 2\, n_{1,\,1} + 6\, n_{2,\,1} + 12\, n_{2,\,2} \tag{18}$$

```
> x2:={'sum(n[i,j],j=1..c1)=m[i]'$'i'=1..c} ;
x3:={'sum(n[i,j],i=1..c1)=f[j]'$'j'=1..c};
```

$$x2 := \{n_{1,\,1} + n_{1,\,2} + n_{1,\,3} = 22,\, n_{2,\,1} + n_{2,\,2} + n_{2,\,3} = 19\}$$
$$x3 := \{n_{1,\,1} + n_{2,\,1} + n_{3,\,1} = 20,\, n_{1,\,2} + n_{2,\,2} + n_{3,\,2} = 20\} \tag{19}$$

```
> x4:=LPSolve(x1,{op(x2),op(x3)},assume={integer,nonnegative},
maximize=true);
```

$$x4 := [268,\, [n_{1,\,1} = 20,\, n_{1,\,2} = 1,\, n_{1,\,3} = 1,\, n_{2,\,1} = 0,\, n_{2,\,2} = 19,\, n_{2,\,3} = 0,\, n_{3,\,1} = 0,\, n_{3,\,2} = 0]] \tag{20}$$

```
> x5:=op(op(2,x4));
```

$$x5 := n_{1,\,1} = 20,\, n_{1,\,2} = 1,\, n_{1,\,3} = 1,\, n_{2,\,1} = 0,\, n_{2,\,2} = 19,\, n_{2,\,3} = 0,\, n_{3,\,1} = 0,\, n_{3,\,2} = 0 \tag{21}$$

```
> x6:=eval(subs(x5,{'''signum(n[i,j]*n[i,j2])*(vm(hm[i],hf[j])+d[i,
j]-vm(hm[i],hf[j2])-d[i,j2])'$'j'=j2..c1'$'j2'=1..c1'$'i'=1..c}))
;
```

$$x6 := \{0,\, -2 + d_{1,\,3} - d_{1,\,2},\, -1 + d_{1,\,3} - d_{1,\,1},\, 1 + d_{1,\,2} - d_{1,\,1}\} \tag{22}$$

```
> x7:=eval(subs(x5,{'''signum(n[j,i]*n[j2,i])*(vf(hm[j],hf[i])-d[j,
i]-vf(hm[j2],hf[i])+d[j2,i])'$'j'=j2..c1'$'j2'=1..c1'$'i'=1..c}))
;
```

$$x7 := \{0,\, 4 - d_{2,\,2} + d_{1,\,2}\} \tag{23}$$

```
> x8:=x6 union x7 minus {0} union {'d[c1,i]=0'$'i'=1..c} union {'d
[i,c1]=0'$'i'=1..c};
```

$$x8 := \{-2 + d_{1,\,3} - d_{1,\,2},\, -1 + d_{1,\,3} - d_{1,\,1},\, 1 + d_{1,\,2} - d_{1,\,1},\, 4 - d_{2,\,2} + d_{1,\,2},\, d_{1,\,3} = 0,\, d_{2,\,3} = 0,\, d_{3,\,1} = 0,\, d_{3,\,2} = 0\} \tag{24}$$

```
> x9:=solve(x8);
x10:=subs(x9,x5,sum(sum(n[i,j]*d[i,j],i=1..c),j=1..c)/sum(sum(n
[i,j],i=1..c),j=1..c));
```

$$x9 := \{d_{1,\,1} = -1,\, d_{1,\,2} = -2,\, d_{1,\,3} = 0,\, d_{2,\,2} = 2,\, d_{2,\,3} = 0,\, d_{3,\,1} = 0,\, d_{3,\,2} = 0\}$$
$$x10 := \frac{2}{5} \tag{25}$$

```
> x11:=[''d[i,j]=0'$i=1..c'$j=1..c];
```

$$x11 := [d_{1,\,1} = 0,\, d_{2,\,1} = 0,\, d_{1,\,2} = 0,\, d_{2,\,2} = 0] \tag{26}$$

```
> x12b:=subs(x11,x5,subs(x9,[[hf[i],sum(n[i,j]*d[i,j],j=1..c)/sum(n
  [i,j],j=1..c)]$i=1..c]));
```

$$x12b := \left[\left[1, -\frac{22}{21}\right], [2, 2]\right] \quad (27)$$

$$x9 := \{d_{1,1} = -1, d_{1,2} = -2, d_{1,3} = 0, d_{2,2} = 0, d_{2,3} = 0, d_{3,1} = 0, d_{3,2} = 0\}$$

$$x10 := -\frac{11}{20} \quad (28)$$

```
> plot([x12a,x12b],color=[red,green]);
```