```
> restart;
> f:=(a,n)->a^alpha*n^(1-alpha);
```

$$f := (a, n) \rightarrow a^{\alpha} n^{1-\alpha} \quad (1)$$

```
> #budget constraint
> x1:=(f(a,nh)-D[2](f)(a,ns)*(nh-n))/n-c;
```

$$x1 := \frac{a^{\alpha} nh^{1-\alpha} - \frac{a^{\alpha} ns^{1-\alpha} (1-\alpha) (nh-n)}{ns}}{n} - c \quad (2)$$

```
> x2:=simplify(isolate(diff(x1,nh),nh),assume=positive);
```

$$x2 := nh = ns \quad (3)$$

```
> #private budget constraint-ns is given
> x3:=subs(x2,x1);
  #social planner budget constraint -ns=n
  x4:=subs(ns=n,x3);
```

$$x3 := \frac{a^{\alpha} ns^{1-\alpha} - \frac{a^{\alpha} ns^{1-\alpha} (1-\alpha) (ns-n)}{ns}}{n} - c$$

$$x4 := \frac{a^{\alpha} n^{1-\alpha}}{n} - c \quad (4)$$

```
> #a utility function for mild complements
  x4a:=(c^(-1/2)+n^(-1/2))^(-2);
```

$$x4a := \frac{1}{\left(\frac{1}{\sqrt{c}} + \frac{1}{\sqrt{n}}\right)^2} \quad (5)$$

```
> #maximize utility fundction given privat budget constraint;
  #choose parameters so that optimal private utility is 3.
  extrema(x4a,{x4},{n,c},'x4b');
  x4b;
  solve(subs(alpha=1/2,3 = (alpha^2*a^(-alpha))^(-1/(1+alpha))),a);
  x4c:=simplify(subs(alpha=1/2,a=27/16,x4b[1]));
```

$$\left\{ \frac{\alpha^2 \left(\alpha^2 a^{-\alpha}\right)^{-\frac{2}{1+\alpha}}}{\left(\sqrt{\left(\alpha^2 a^{-\alpha}\right)^{-\frac{1}{1+\alpha}}} + \sqrt{\alpha^2 \left(\alpha^2 a^{-\alpha}\right)^{-\frac{1}{1+\alpha}}}\right)^2} \right\}$$

$$\left\{ \left\{ c = \alpha^2 \left(\alpha^2 a^{-\alpha}\right)^{-\frac{1}{1+\alpha}}, n = \left(\alpha^2 a^{-\alpha}\right)^{-\frac{1}{1+\alpha}} \right\} \right\}$$

$$\frac{27}{16}$$

$$x4c := \left\{ c = \frac{3}{4}, n = 3 \right\} \quad (6)$$

```
> #slope of private budget constraint evaluated at ns=n
> x5:=simplify(diff(x3,n)/diff(x3,c));
  x5a:=simplify(subs(ns=n,x5));
```

$$x5 := \frac{a^{\alpha}\, ns^{1-\alpha}\, \alpha}{n^2}$$

$$x5a := n^{-1-\alpha}\, a^{\alpha}\, \alpha \tag{7}$$

```
> #slope of social plannter budget constraint
  x6:=simplify(diff(x4,n)/diff(x4,c));
```

$$x6 := n^{-1-\alpha}\, a^{\alpha}\, \alpha \tag{8}$$

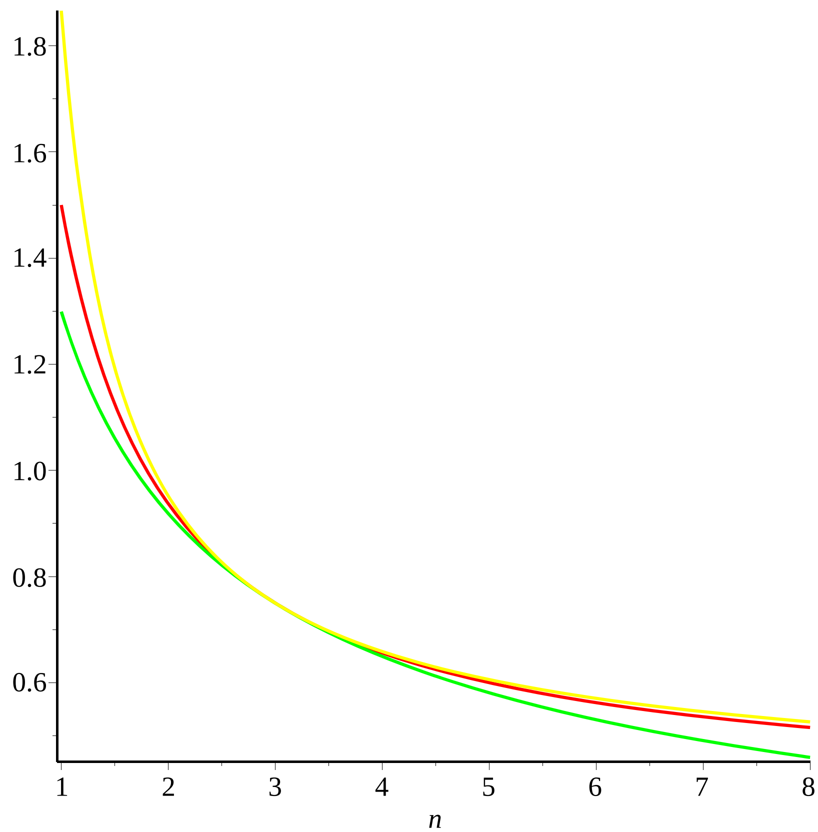
```
> parms:=alpha=1/2, a=27/16;
```

$$parms := \alpha = \frac{1}{2},\ a = \frac{27}{16} \tag{9}$$

```
> x7:=subs(x4c,x4a);
```

$$x7 := \frac{1}{\left(\frac{1}{3}\sqrt{3}\,\sqrt{4} + \frac{1}{3}\sqrt{3}\right)^2} \tag{10}$$

```
> #plot of social budget contraint (green), and private budget
  constraint (red), at private fertility optimum;
> plot(subs(parms,ns=3,[solve(x3,c),solve(x4,c),solve(x7=x4a,c)[1]]
  ),n=1..8,color=[red,green,yellow]);
```



1.8
1.6
1.4
1.2
1.0
0.8
0.6
1
2
3
4
5
6
7
8
n