```
> restart;
> #u1 is utility from store 1; u2 is utility from store 2;
> u1:=v1+e1;
  u2:=v2+e2;
```

$$u1 := v1 + e1$$
$$u2 := v2 + e2 \quad \textbf{(1)}$$

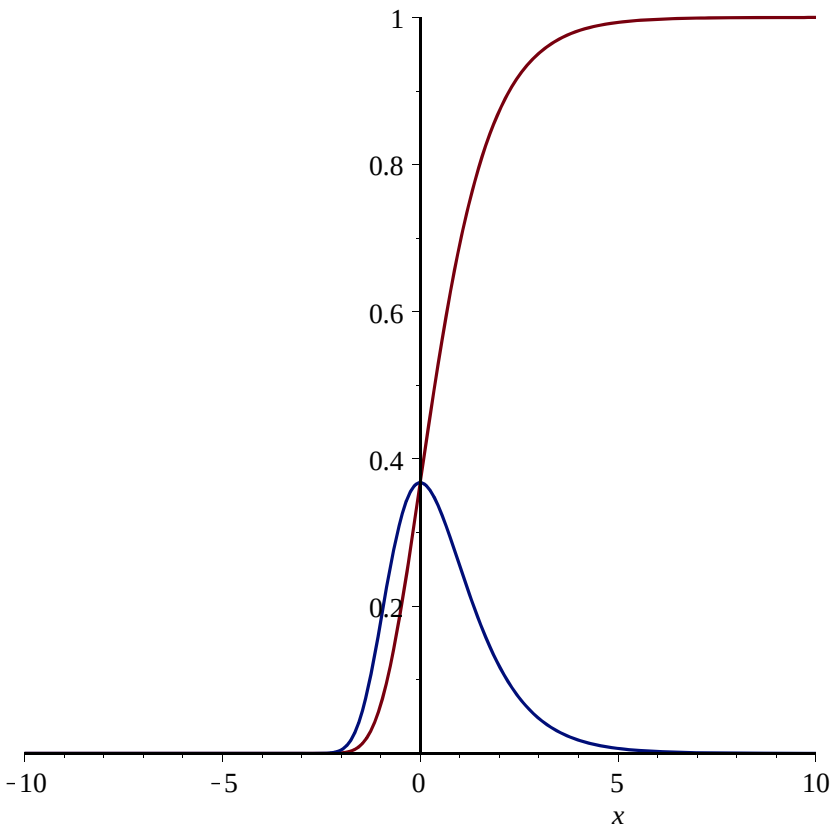
```
> #distribution of errors;
> F:=x->exp(-exp(-x));
```

$$F := x \rightarrow e^{-e^{-x}} \quad \textbf{(2)}$$

```
> #density of errors
> f:=x->diff(F(x),x);
```

$$f := x \rightarrow \frac{d}{dx} F(x) \quad \textbf{(3)}$$

```
> plot([F(x),f(x)]);
```




```
> #Prob(u2>u1)=Prob(e2<v1+e1-v2)
> d1:=simplify(int(F(v1+e1-v2)*f(e1),e1=-infinity..infinity),
```

```
assume=positive);
d2:=simplify(1-d1);
```

$$d1 := \frac{e^{v1}}{e^{v2} + e^{v1}}$$

$$d2 := \frac{e^{v2}}{e^{v2} + e^{v1}} \tag{4}$$

```
> #this formulat describes d1
> #Now let utility v1=q1-p1
> v1:=q1-p1;
  v2:=q2-p2;
```

$$v1 := q1 - p1$$

$$v2 := q2 - p2 \tag{5}$$

```
> #profits for two firms are price times quantity minus the cost of
  quality
> x1:=p1*d1-c1*q1^2+c3*q1*q2;
  x2:=p2*d2-c2*q2^2+c3*q1*q2;
```

$$x1 := \frac{p1\, e^{q1 - p1}}{e^{q2 - p2} + e^{q1 - p1}} - c1\, q1^2 + c3\, q1\, q2$$

$$x2 := \frac{p2\, e^{q2 - p2}}{e^{q2 - p2} + e^{q1 - p1}} - c2\, q2^2 + c3\, q1\, q2 \tag{6}$$

```
> #first order conditions--each store maximizes price and quality
  given other firm's behavior
> x3:={diff(x1,p1),diff(x1,q1),diff(x2,p2),diff(x2,q2)};
```

$$x3 := \left\{ \frac{e^{q1 - p1}}{e^{q2 - p2} + e^{q1 - p1}} + \frac{p1\, \left(e^{q1 - p1}\right)^2}{\left(e^{q2 - p2} + e^{q1 - p1}\right)^2} - \frac{p1\, e^{q1 - p1}}{e^{q2 - p2} + e^{q1 - p1}}, \frac{e^{q2 - p2}}{e^{q2 - p2} + e^{q1 - p1}} - \frac{p2\, e^{q2 - p2}}{e^{q2 - p2} + e^{q1 - p1}} + \frac{p2\, \left(e^{q2 - p2}\right)^2}{\left(e^{q2 - p2} + e^{q1 - p1}\right)^2}, -\frac{p1\, \left(e^{q1 - p1}\right)^2}{\left(e^{q2 - p2} + e^{q1 - p1}\right)^2} + \frac{p1\, e^{q1 - p1}}{e^{q2 - p2} + e^{q1 - p1}} - 2\, c1\, q1 + c3\, q2, \frac{p2\, e^{q2 - p2}}{e^{q2 - p2} + e^{q1 - p1}} - \frac{p2\, \left(e^{q2 - p2}\right)^2}{\left(e^{q2 - p2} + e^{q1 - p1}\right)^2} - 2\, c2\, q2 + c3\, q1 \right\} \tag{7}$$

```
> #a little program to solve x3;
> fs:=proc(c12,c11,c13);
  x4:=subs(c2=c12,c1=c11,c3=c13,x3);
  x5:=fsolve(x4);
  x6:=subs(x5,[c12,p1,p2,q1,q2]);

  end;
Warning, `x4` is implicitly declared local to procedure `fs`
Warning, `x5` is implicitly declared local to procedure `fs`
Warning, `x6` is implicitly declared local to procedure `fs`
```

```
fs := proc(c12, c11, c13)
    local x4, x5, x6;
    x4 := subs(c2 = c12, c1 = c11, c3 = c13, x3);
    x5 := fsolve(x4);
    x6 := subs(x5, [c12, p1, p2, q1, q2])
end proc
```

(8)

```
> #solve price and quality for c3=0,c1=1
> fsmat:=['fs(c/10,1,0)'$'c'=1..10];
  #solve price and quality for c3=.1,c1=1
  fsmat2:=['fs(c/10,1,.1)'$'c'=1..10];
```

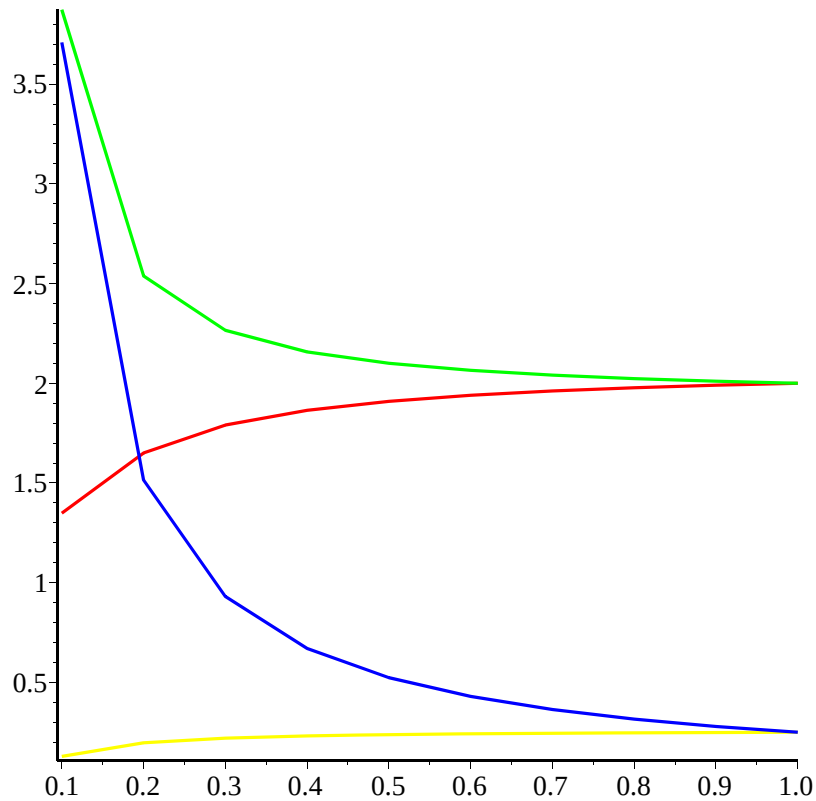
$$fsmat := \left[\left[\frac{1}{10}, 1.348109580, 3.872658661, 0.1291102686, 3.708897314\right], \left[\frac{1}{5}, 1.650279477, 2.537800339, 0.1970210155, 1.514894923\right], \left[\frac{3}{10}, 1.790349982, 2.265262254, 0.2207249951, 0.9309166829\right], \left[\frac{2}{5}, 1.864250169, 2.157072380, 0.2317956526, 0.6705108686\right], \left[\frac{1}{2}, 1.909265154, 2.099789204, 0.2381191402, 0.5237617195\right], \left[\frac{3}{5}, 1.939444076, 2.064459317, 0.2421941648, 0.4296763920\right], \left[\frac{7}{10}, 1.961052001, 2.040526422, 0.2450348080, 0.3642359886\right], \left[\frac{4}{5}, 1.977275274, 2.023253147, 0.2471267625, 0.3160915468\right], \left[\frac{9}{10}, 1.989899211, 2.010203856, 0.2487309924, 0.2791877863\right], [1, 2.000000000, 2.000000000, 0.2500000000, 0.2500000000]\right]$$

(9)

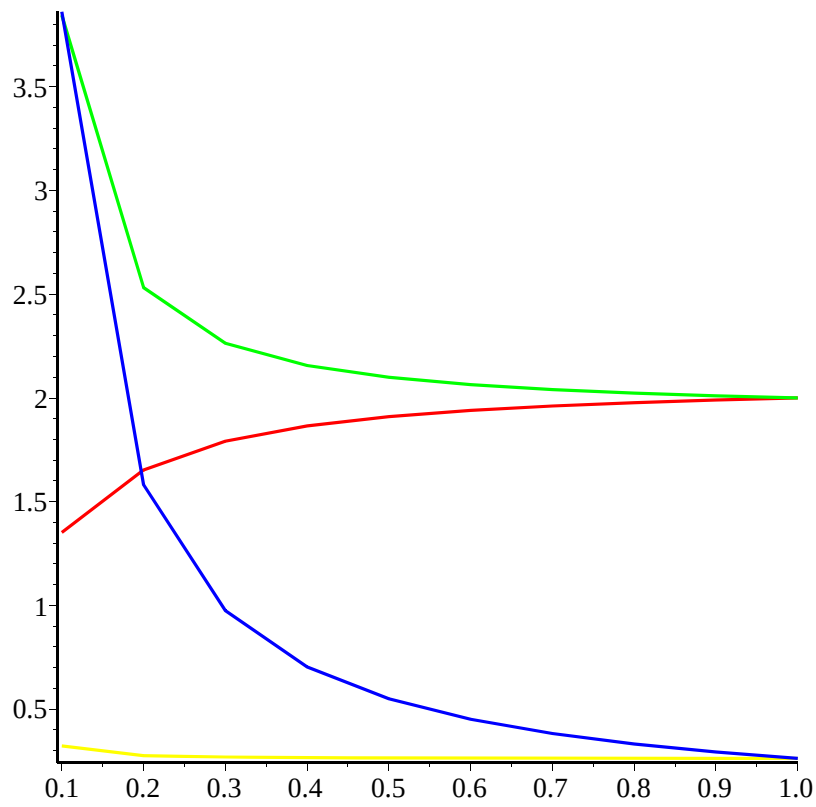
$$fsmat2 := \left[\left[\frac{1}{10}, 1.351604249, 3.844106703, 0.3231124258, 3.860863910\right], \left[\frac{1}{5}, 1.652900962, 2.531625864, 0.2765834461, 1.581638175\right], \left[\frac{3}{10}, 1.791547369, 2.263348271, 0.2696736551, 0.9752401106\right], \left[\frac{2}{5}, 1.864852145, 2.156267005, 0.2670661138, 0.7036776912\right], \left[\frac{1}{2}, 1.909591119, 2.099395079, 0.2656758379, 0.5502398977\right], \left[\frac{3}{5}, 1.939626359, 2.064252818, 0.2648035416, 0.4517029737\right], \left[\frac{7}{10}, 1.961152755, 2.040417348, 0.2642023504, 0.3830888725\right], \left[\frac{4}{5}, 1.977326779, 2.023199222, 0.2637617705, 0.3325684240\right], \left[\frac{9}{10}, 1.989919573, 2.010183076, 0.2634245449, 0.2938196263\right], [1, 2.000000000, 2.000000000, 0.2631578947,$$

0.2631578947]

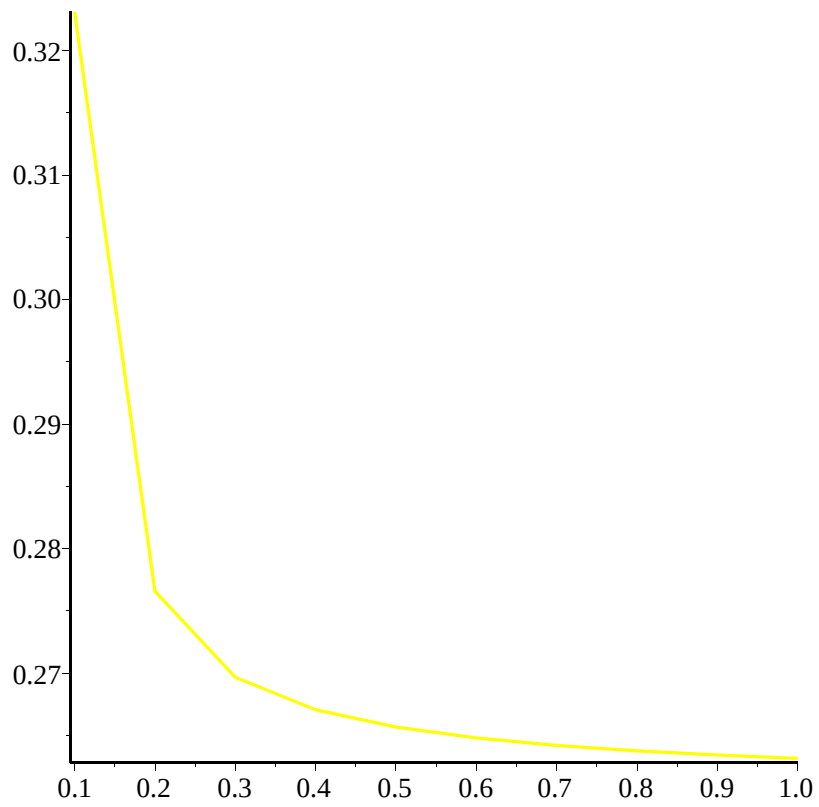
```
> plot([fsmat[..,[1,2]],fsmat[..,[1,3]],fsmat[..,[1,4]],fsmat[..,
  [1,5]]],color=[red,green,yellow,blue]);
```




```
> plot([fsmat2[..,[1,2]],fsmat2[..,[1,3]],fsmat2[..,[1,4]],fsmat2[.
  .,[1,5]]],color=[red,green,yellow,blue]);
```




```
> plot([fsmat2[..,[1,4]]],color=[yellow]);
```


0.32
0.31
0.30
0.29
0.28
0.27
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0