Research Report

A VISUAL EQUALIZATION STRATEGY FOR LOCOMOTOR CONTROL: Of Honeybees, Robots, and Humans

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Abstract—*Honeybees fly down the center of a corridor by equating the speed of optic flow in the lateral field of the two eyes. This flow-equalization strategy has been successfully implemented in mobile robots to guide behavior in cluttered environments. We investigated whether humans use a similar strategy to steer down a corridor, and determined the relative contributions of equating the speed of flow (.27), the splay angles of base lines (.62), and the visual angles of texture on the left and right walls (.03) to steering behavior. A generalized equalization strategy based on the weighted linear combination of these variables closely models human behavior, providing robust visual control.*

The control of locomotion involves several component tasks that are essential to any mobile agent, whether human, animal, or robot. They include steering toward goals, down straight passageways, and around curved ones, while simultaneously avoiding obstacles. Such locomotor behaviors may be governed by laws of visual control that map relevant optical information into action variables (Gibson, 1958; Warren, 1998b). We have recently found that a linear combination of optic flow and egocentric direction is used to guide walking to a goal (Warren, Kay, Zosh, Duchon, & Sahuc, 2001). In the present experiments, we investigated how multiple optical variables contribute to the task of steering down a passageway.

Srinivasan and his colleagues (Srinivasan, Lehrer, Kirchner, & Zhang, 1991) have shown that honeybees (*Apis mellifera*) fly down the center of a corridor or through an opening by equating the speed of optic flow in the lateral portion of the two eyes. If motion is artificially added to one of the side walls, thereby increasing the flow speed on that side, the bees fly away from the moving wall to the *balance point* at which the flow speeds are equal (Fig. 1), apparently ignoring other distance information. We have implemented versions of this strategy on a mobile robot platform in a cluttered environment (Duchon & Warren, 1994; Duchon, Warren, & Kaelbling, 1998; see also Coombs, Herman, Hong, & Nashman, 1998; Santos-Victor, Sandini, Curotto, & Giribaldi, 1995; Weber, Venkatesh, & Srinivasan, 1997). Flow equalization proves to be a robust control strategy, guiding the robot down passages, through openings, and around obstacles without relying on an internal world model. As an obstacle is approached, for example, its flow speed increases and the robot steers away from it. The simplicity and generality of flow equalization suggest that it might provide a basic locomotor control law in humans as well.

Although optic flow is a likely source of information (Warren, 1998a), a number of other potential variables are also available, particularly in man-made environments such as hallways and roadways. The base line where a wall meets the floor, the edge of a road, or a lane marker projects in the image plane at an angle with respect to the vertical, known as the *splay angle*. Splay angle is known to stabilize steering in a driving simulator (Beal & Loomis, 1996; Land & Horwood, 1995), and equalizing the left and right splay angles would allow one to stay in the middle of a straight corridor. Another candidate is the visual spatial frequency of wall texture, which we call *texture scale*. If the texture on both walls is statistically similar, one could stay in the middle of a corridor by equating the optical texture scale on the left and right walls. Other variables that could be equalized in a similar manner include binocular disparities and optical texture gradients for the walls.

We tested the role of flow speed, splay angle, and texture scale by asking participants to steer down the center of a corridor in a virtual environment, either using a joystick or walking on a treadmill. By manipulating the optical information, we could specify the balance point for each variable independently: (a) Flow speed was manipulated by adding longitudinal motion to the left or right side wall, opposite the direction of the observer's travel. (b) Texture scale was manipulated by varying the absolute size of the texture on the left or right wall. (c) Splay angle was manipulated by introducing or removing base lines for the side walls, which created an implicit floor in the corridor (see Fig. 2). The equalization strategy predicts that the participant will steer to the balance point for a given variable. Placing these variables in conflict allowed us to determine their relative contributions and develop a simple model of steering behavior.

EXPERIMENT 1

In the first experiment, we tested the flow-equalization strategy and its interaction with splay angle. Participants used a joystick to steer down the virtual corridor while the time series of lateral position was recorded.

Method

Interactive displays of self-motion down a textured corridor were generated on a Silicon Graphics Crimson RE at a frame rate of 60 Hz. The corridor was 9.6 m wide, with an eye height of 1.6 m. The wall texture was based on a Voronoi tessellation. Patches were defined by determining a set of random seed points on the surface, assigning them a random RGB (red, green, blue) value, and setting the color of other points to be the same as the nearest seed point. Irregular boundaries were created by adding noise to the distance from the seed point. To simulate forward self-motion, the walls moved longitudinally at a constant "observer speed" ($\dot{z}_0$ = 12.8 m/s). Either the speeds of the two walls were the same ($1.0\dot{z}_0$), with a balance point at the corridor center (0 m), or motion was added to one wall so that it moved 1.5 times as fast as the other wall ($1.5\dot{z}_0$), with a balance point 0.96 m from the center, or twice as fast as the other wall ($2.0\dot{z}_0$), with a balance point 1.60 m from the center. The three wall-speed conditions were crossed with two splay conditions (splay or no splay). The dis-

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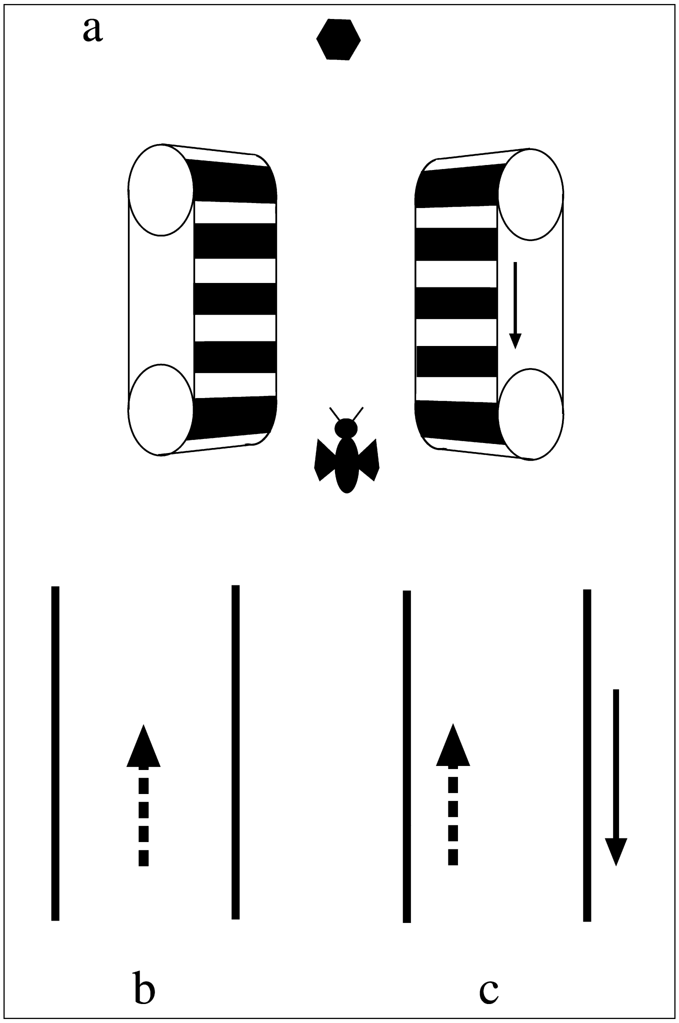

Fig. 1. Honeybee experiments of Srinivasan, Lehrer, Kirchner, and Zhang (1991). Bees were trained to fly down a corridor toward a feeder (a). The walls were rubber sheets on motorized rollers so that the stripes on one wall could be moved independently. When there was no motion (b), the bees tended to fly down the middle of the corridor. When the right wall moved opposite the direction of the bees' motion (c), the bees flew left of the center to a position at which the flow rates from the two walls were equal.

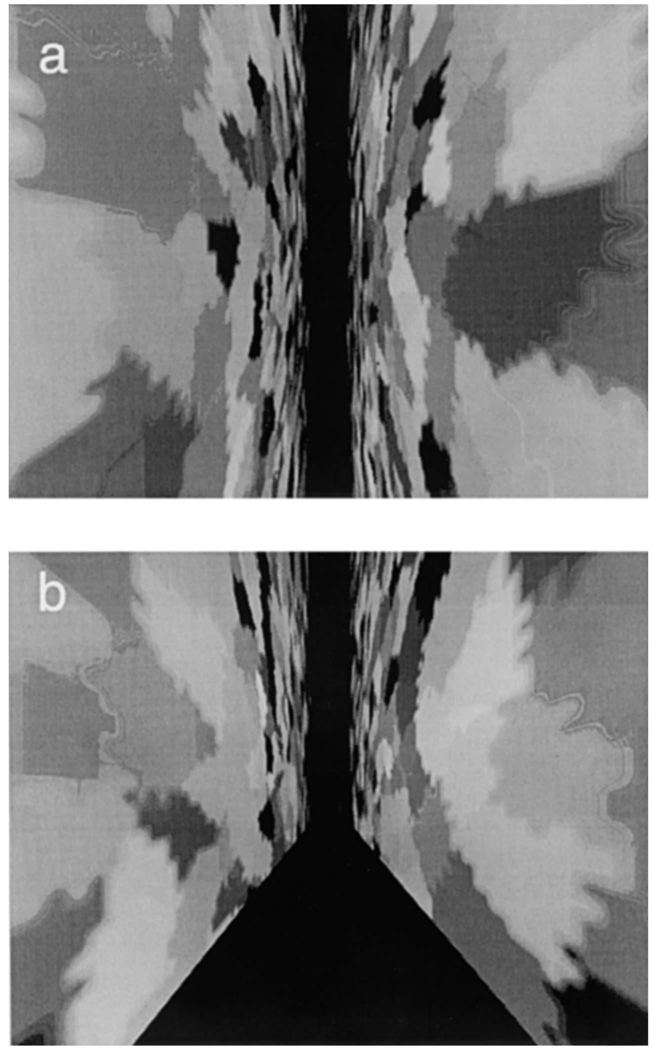

Fig. 2. Corridor displays (a) without a floor (no splay) and (b) with an implicit floor (splay). The corridor width is 2.4 m, as in Experiments 2 and 3.

plays were presented on a monitor at a resolution of 640 × 496 pixels and viewed monocularly through an elliptical mask (visual angle of 40° horizontal, H, × 33° vertical, V). At the beginning of a trial, participants started out near the center of the corridor and were instructed to steer down the middle as best they could. The left/right position of the joystick controlled the lateral component of observers' velocity, and a trial lasted 15 s. Eight participants each performed 16 trials per condition in a randomized order, blocked by splay condition.

Results and Discussion

The mean time series of lateral position for all participants appear in Figure 3, plotted so the moving wall is on the right. We statistically analyzed the mean position of the last 5 s from each trial. Participants systematically steered away from the moving wall, $F(2, 14) = 17.12$, $p < .001$, a result consistent with the flow-equalization strategy. The addition of splay angle also had a strong influence, $F(1, 7) = 11.34$, $p < .02$, attenuating but not eliminating the effect of wall speed. In the no-splay condition, the final position for a wall speed of $1.5\dot{z}_0$ was -0.82 m, about 85% of the distance to the predicted flow balance point; with a wall speed of $2.0\dot{z}_0$, the final position was -1.17 m, about 70% of the way to the predicted position. The same overall pattern was present in the splay condition, but the final positions were reduced by about 60%, Wall Speed × Splay interaction, $F(2, 14) = 7.32$, $p < .01$. Although we did not directly test equalization of splay angle by manipulating the base lines, steering was strongly biased toward the splay balance point at 0 m, and one splay angle by itself was insufficient to specify this position. Thus, both equalization of flow speed and equalization of splay angle appear to contribute to steering control, with the latter having a stronger influence.

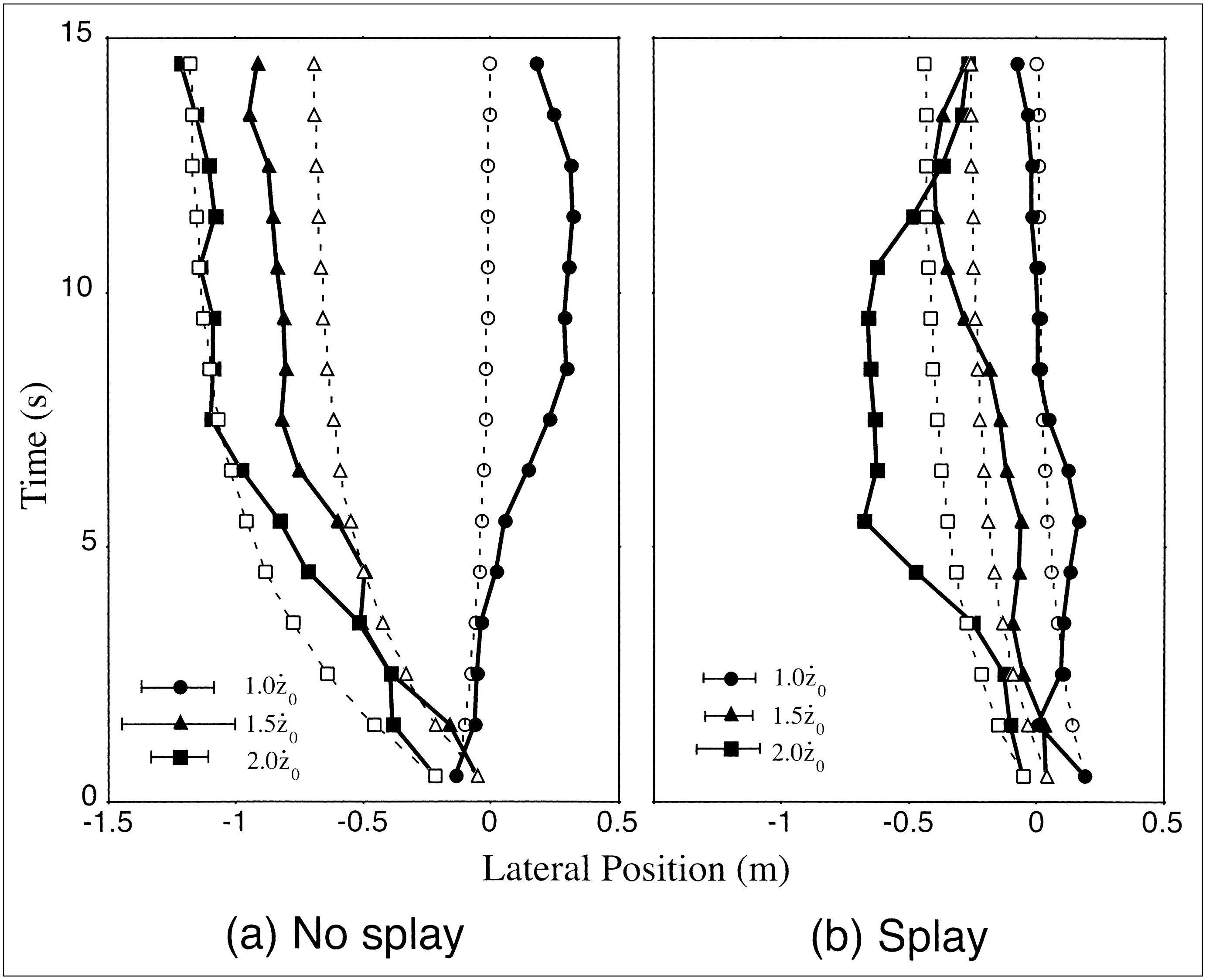

Fig. 3. Mean time series of lateral position in the corridor for all participants in Experiment 1 (joystick). Data are plotted with the moving wall on the right. The two panels show results for displays without (a) and with (b) splay for three different speeds of the moving wall relative to observer speed ($\dot{z}_0$). Closed symbols show the participants' data, and open symbols show the results of the model simulation. Error bars indicate the mean subject standard error (± 1 *SE*) over time in each condition.

EXPERIMENT 2

To determine whether the equalization strategy would generalize to more natural locomotion, we repeated the first experiment with participants walking on a treadmill in front of a large display. One group of participants was instructed to walk in the middle of the corridor, as before, and another group was just told to walk normally. We also tested two observer speeds to vary the overall flow rate.

Method

Displays were the same as before, except that the corridor was narrower (2.4 m) and observer speed was slower. The same three wall-speed ($1.0\dot{z}_0$, $1.5\dot{z}_0$, and $2.0\dot{z}_0$) and two splay conditions were tested, crossed with two base observer speeds, $\dot{z}_0$, of 1 m/s (the actual treadmill speed) and 2 m/s. The motion balance points were -0.24 m with a wall speed of $1.5\dot{z}_0$ and -0.40 m with a wall speed of $2.0\dot{z}_0$. Participants walked on a wide-body (122 cm wide × 183 cm long) treadmill while viewing a rear-projection screen (3.0 m H × 2.2 m V) from a distance of about 1 m. The center of projection was updated in real time by a Polhemus electromagnetic sensor worn on the subject's forehead. The display was viewed monocularly through a head-mounted mask (field of view was 90° H × 70° V). One group of 12 participants was instructed to "move to the middle of the hallway and stay there throughout the duration of the trial"; the second group of 12 was asked to "walk down the hallway as you normally would—to a position in the hallway which is comfortable." There were 14 trials per condition, each lasting 15 s, blocked by observer speed.

Results and Discussion

There was no main effect or interactions involving instruction condition (all $ps > .9$). Figure 4 shows the mean time series at the base speed of 2 m/s when the instructions were to "walk in the middle." Once again, participants steered away from the moving wall, $F(2, 44) = 28.66$, $p < .001$, a pattern consistent with flow equalization, and this result was attenuated by splay angle, $F(1, 22) = 150.28$, $p < .001$. Lateral deviations were greater for the higher than for the lower observer speed, $F(1, 22) = 12.57$, $p = .002$. Participants moved about two thirds of the way to the predicted positions at both wall speeds, but this effect was reduced by about 60% in the splay condition, Wall Speed × Splay interaction, $F(2, 44) = 47.076$, $p < .001$. Thus, the previous pattern of results was replicated for actual walking, even without explicit instructions to walk down the middle of the corridor.

EXPERIMENT 3

In the final experiment, we added texture-scale information and compared its effects with those of flow speed and splay angle during treadmill walking.

Method

The displays and equipment were similar to those in Experiment 2. The absolute texture size on the moving wall was manipulated so that

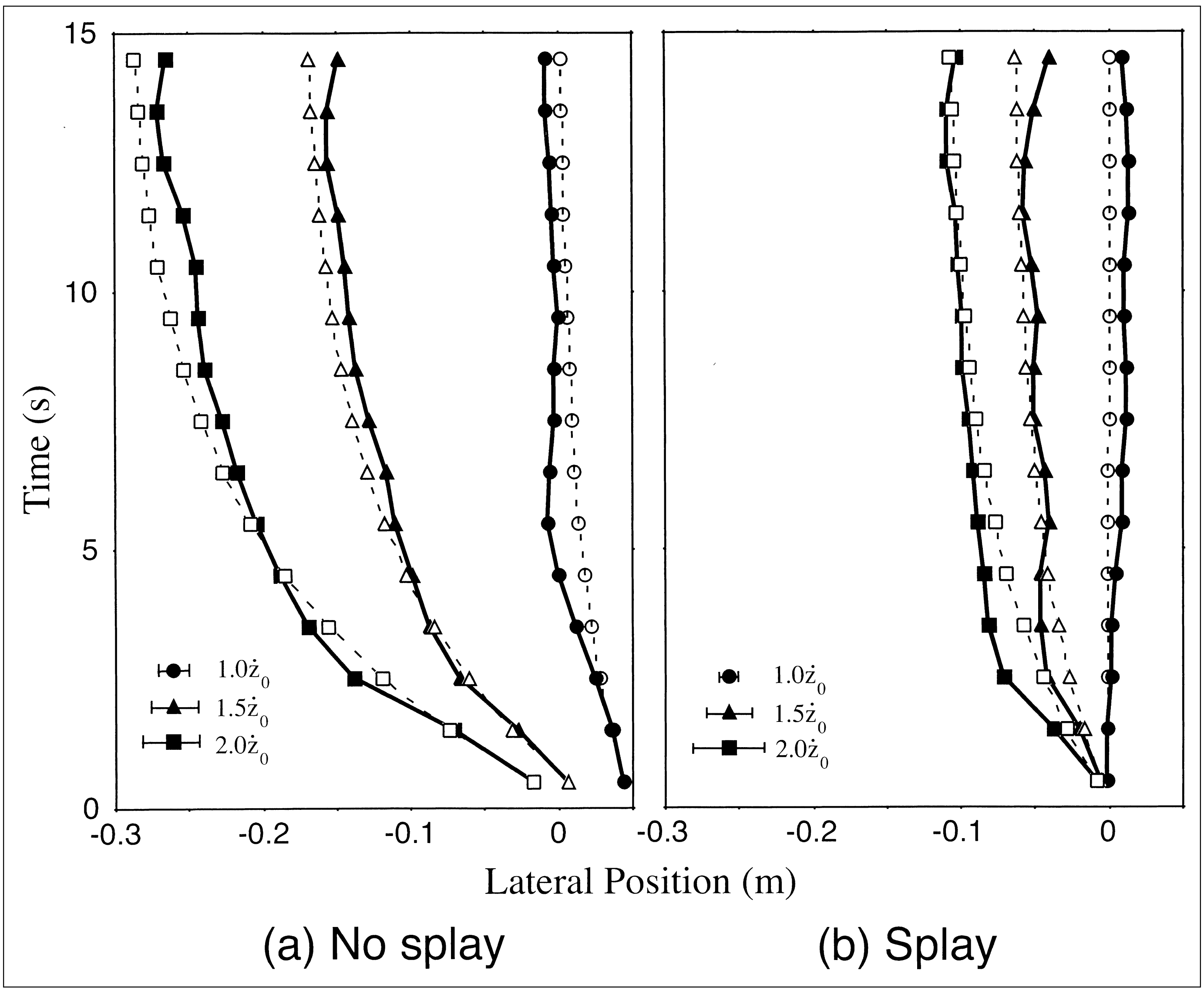

Fig. 4. Mean time series of lateral position in the corridor for all participants instructed to walk in the "middle of the hallway" in Experiment 2 (treadmill). Data are plotted with the moving wall on the right. The two panels show results for displays without (a) and with (b) splay for three different speeds of the moving wall relative to observer speed ($\dot{z}_0$), for trials on which $\dot{z}_0$ was 2 m/s. Closed symbols show the participants' data, and open symbols show the results of the model simulation. Error bars indicate the mean subject standard error (±1 *SE*) over time in each condition.

it was equal to that on the opposite wall ($1.0t_0$), half the size ($0.5t_0$), or twice the size ($2.0t_0$). Texture size was crossed with two wall speeds ($1.0\dot{z}_0$ and $2.0\dot{z}_0$) and the two splay conditions, at one base observer velocity ($\dot{z}_0$ = 2 m/s). Thirteen participants were instructed to walk down the middle of the corridor. They performed 14 trials in each condition, in a randomized order.

Results and Discussion

The mean time series for all participants appear in Figure 5. Participants reliably walked away from the side with the larger texture, $F(2, 24) = 9.937$, $p < .001$, but the effect was quite small and decreased by about 60% when splay was present, Texture × Splay interaction, $F(2, 24) = 16.372$, $p < .001$. There was also a significant Texture × Splay × Wall Speed interaction, $F(2, 24) = 4.016$, $p < .05$. As before, the main effects of splay, $F(1, 12) = 150.801$, $p < .001$, and wall speed, $F(1, 12) = 226.208$, $p < .001$, as well as the Splay × Wall Speed interaction, $F(1, 12) = 72.794$, $p < .001$, were significant. These data suggest that texture equalization plays a small but reliable role in steering control.

GENERAL DISCUSSION

The results indicate that splay angle, flow speed, and texture scale all contribute to a generalized equalization strategy for steering, with decreasing influence respectively. Other variables, such as texture gradients and binocular disparity, also specify the observer's position in the corridor and may be exploited in a similar manner. We thus derived a simple dynamic control law (Schöner, Dose, & Engels, 1995; Warren, 1998b) in which the rate of change in lateral position ($\dot{x}$) is a

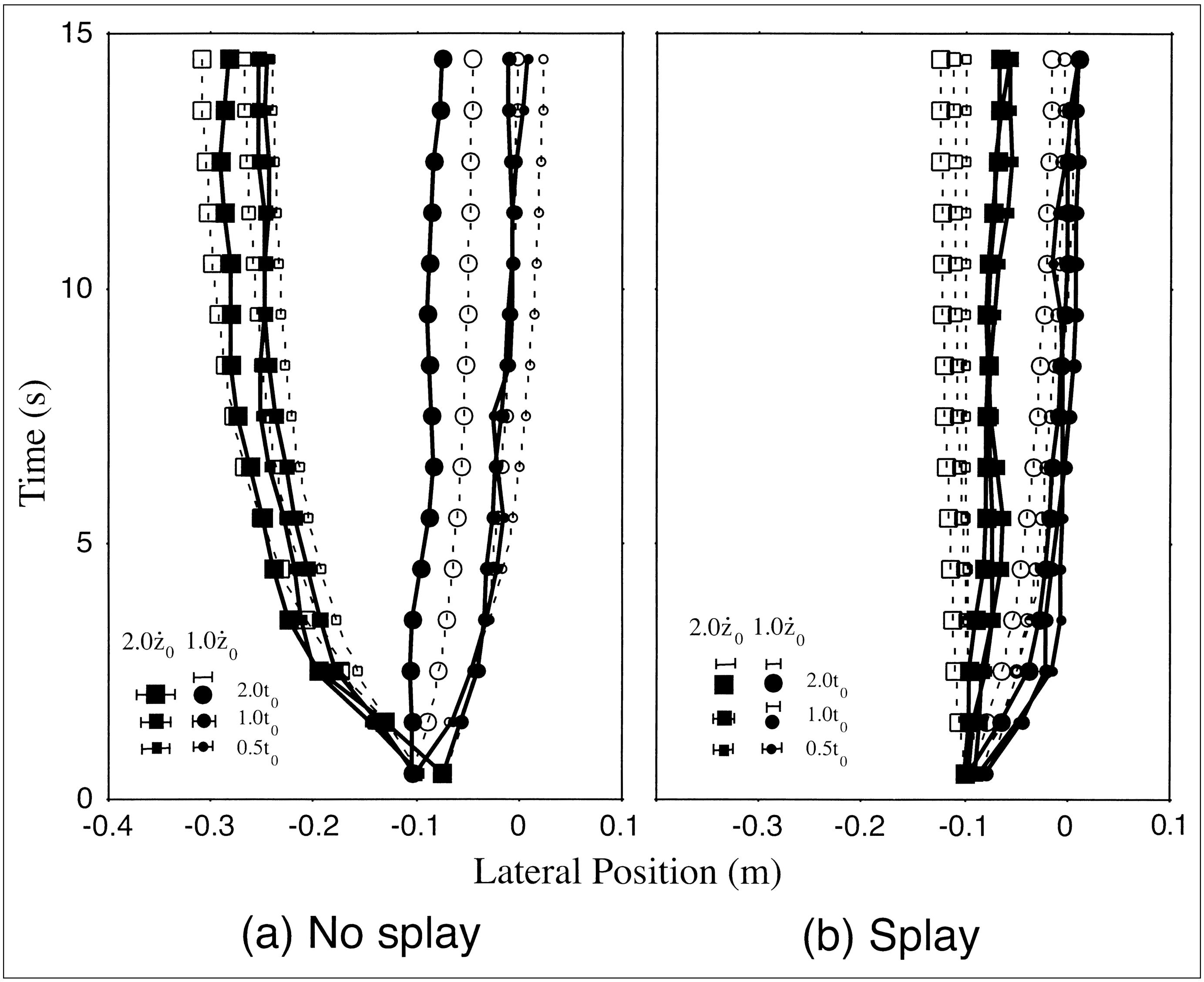

Fig. 5. Mean time series of lateral position in the corridor for all participants in Experiment 3. Data are plotted with the moving wall on the right. The two panels show results for displays without (a) and with (b) splay for two different speeds of the moving wall relative to observer speed ($\dot{z}_0$) and for three different texture scales on the moving wall relative to the opposite wall (t_0), indicated by different symbol sizes. Closed symbols show the participants' data, and open symbols show the results of the model simulation.

function of the current position (x) specified by a weighted (w) linear combination of optical variables:

$$\dot{x} = -\mathrm{k}(w_s x_s + w_f x_f + w_t x_t + w_m x_m). \quad (1)$$

Each term represents the current distance x from the balance point (in units of corridor width) specified by the normalized difference between left (L) and right (R) optical variables. An advantage of this formulation is that behavior is scale invariant, independent of corridor width. The splay term is

$$x_s = \left(\frac{\tan\varphi_L - \tan\varphi_R}{\tan\varphi_L + \tan\varphi_R}\right)0.5, \quad (2)$$

where φ represents splay angle with respect to the vertical. The flow term is

$$x_f = \left(\frac{\dot{\beta}_R - \dot{\beta}_L}{\dot{\beta}_R + \dot{\beta}_L}\right)0.5, \quad (3)$$

where $\dot{\beta}$ is flow speed (sampled at the same angle on the left and right wall). The texture-scale term is

$$x_t = \left(\frac{\tan\alpha_R - \tan\alpha_L}{\tan\alpha_R + \tan\alpha_L}\right)0.5, \quad (4)$$

where α is the visual angle of an average texture patch (sampled at the same angle on the left and right wall). Finally, x_m is a placeholder for miscellaneous information that specifies the true center of the corridor and could have been equalized on the left and right, such as texture gradients.

The coefficient k, calculated as $c\dot{z}_0$, is a steering-rate constant that determines how quickly the observer moves toward the balance point, by scaling lateral velocity ($\dot{x}$) to the forward velocity ($\dot{z}_0$) with a constant (c). The steering-rate constant is equivalent to a maximum allowed heading deviation from the longitudinal axis of the corridor ($\dot{x}/\dot{z}_0 = \tan \beta$), which is specified optically. The default balance point (in the absence of wall manipulations) is the middle of the corridor, but this could be set to other positions with an additional parameter in Equations 2 through 4.

We simulated the data from all the experiments with this control law, using a fixed set of weights found by gradient descent on the root mean square (RMS) error ($r^2 = .91$, $RMS = 7.51$ cm). The results are represented by the open symbols in Figures 3 through 5. For nearly all the treadmill data points (Figs. 4 and 5), the predicted position is within the standard error of the mean subject data, and the pattern of results is virtually identical to human performance. The resulting weights (which sum to 1.0) are $w_s = .624$ for splay angle, $w_f = .273$ for flow speed, $w_t = .031$ for texture scale, and $w_m = .071$ for any remaining information. The steering-rate constant, k, was $0.249\dot{z}_0$, corresponding to a maximum allowed heading deviation of $\beta = 7.1°$. Thus, when base lines are available, splay angle contributes to maintaining lateral position with a weight more than twice that of the flow speed. Texture scale also influences steering, although its weight is only 1/10 that of the flow speed.

Most of the high RMS error in the data set comes from the joystick experiment (Fig. 3), in which the corridor width was 4 times that in the treadmill experiments. Nevertheless, the pattern of results in the joystick experiment is similar to the pattern of results in the other experiments, suggesting that the same control law governed joystick steering and legged walking. In addition, the lack of difference between instruction groups in Experiment 2 suggests that the model can account for data whether or not subjects consciously try to stay in the middle of the corridor, so the control law may pertain to unconstrained everyday walking.

A similar influence of flow speed was independently found by Chatziastros, Wallis, and Bülthoff (in press), in experiments on steering down a corridor with a mouse controller. However, their results showed some differences from the present findings: The magnitude of the effect was about half that of Experiment 1, there was an equivalent influence of spatial frequency, and the influence of splay angle was not significant. The reduced effect of flow relative to spatial frequency in their study could have been due to using wide vertical stripes on the walls (5 and 10 m in width), which may have provided a weaker motion signal, and the lack of a splay-angle effect could have been due to using a pair of horizontal lines on the walls rather than a more salient virtual floor.

Such effects of wall motion appear to be contrary to a previous result concerning perceived heading, that is, judgment of the direction of self-motion. Dyre and Andersen (1996) reported that perceived heading in a random-dot cloud was biased away from higher flow velocities on one side, which made the path of self-motion appear curved. This finding might lead one to expect compensatory steering toward the higher-velocity moving wall, rather than away from it, in our experiments. Our results are thus consistent with use of an equalization strategy rather than a heading strategy to maintain lateral position in a corridor. It is also possible that asymmetrical flow velocities induce the perception of a curved path in a structureless dot cloud, but not in a straight corridor, where the walls provide a constant reference.

It thus appears that humans, like bees, steer down passageways by equating the speed of optic flow, but also take advantage of other optical variables in a generalized equalization strategy. These findings suggest that simple flow equalization may provide a phylogenetically primitive platform upon which more sophisticated steering strategies were erected. We have recently found that flow equalization sums linearly with a heading strategy during steering around an obstacle in a corridor (Duchon & Warren, 1998). In addition, the heading strategy sums linearly with egocentric direction during steering toward a goal (Rushton, Harris, Lloyd, & Wann, 1998; Warren et al., 2001). Such additive combinations of redundant optical variables may form the basis of a general architecture for visual control laws that are robust under varying environmental conditions.

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