THE FORCE-DRIVEN HARMONIC OSCILLATOR AS A MODEL FOR HUMAN LOCOMOTION *

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The study was conducted to determine whether the preferred frequency of locomotion was predictable as the least amount of energy required to drive a harmonic oscillator. Subjects were instructed to walk at a preferred rate under conditions where their ankles were unloaded and bilaterally loaded. These results were compared to the frequency which was predicted from the formula for a force-driven harmonic oscillator. The length of a simple pendulum equivalent of the lower extremity (thigh, shank, foot, and added mass) was used to approximate the length of the oscillator for prediction purposes. Results indicated that a constant of 2 applied to the gravitational constant of the period prediction formula provided a more accurate representation of the actual frequency. Similar results have been found in the walking gait of quadrupeds of widely varying sizes (Kugler and Turvey 1987).

Introduction

The notion that humans may be self-optimizing machines is supported by research findings in exercise physiology and biomechanics (Wilke 1977). A self-optimizing machine may be conceived as one in which the biological system becomes coordinated in a variety of ways to produce the ultimate in performance at a minimal energy cost. Bipedal gait appears to have developed kinetically and structurally to

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become a metabolically and mechanically efficient system of transportation. For example, anatomical structure and function have ensured that vertical displacement of the center of gravity and inertial changes have been minimized to reduce metabolic energy requirements (Fisher and Gullickson 1978). In addition, the minimal energy criterion applies also to the dynamics of the act of ambulating. The self-selection of stride frequency results in a minimal metabolic energy cost (Cotes and Meade 1960; Zarrugh et al. 1974; Workman and Armstrong 1986). Wheelchair locomotion (Sargent and Van der Woude 1988), manual tire pumping (Corlett and Mahaveda 1970) and arm ergometry (Salvendy and Pilitsis 1971) tasks have also produced a minimal cost when performed at a freely chosen frequency. When subjects are required to perform at frequencies above and below the preferred frequency, the curve of metabolic cost (O_2 consumption) against frequency is V-shaped with the minimum at the preferred frequency (Zarrugh and Radcliffe 1978).

The idea that animal and human behaviors be considered complex oscillatory processes is one which has led to the application of the physics of pendulums to such systems. Cyclicity in a biological system takes place in all components and at all levels (Iberall 1970, 1978) from intracellular replication of DNA, to heart rate and respiration, to electrical activity in the cerebral cortex, to locomotion, circadian rhythmicities, and beyond.

Although biological systems are non-linear and produce limit cycles, the type of oscillation observed in such systems is, within certain boundaries, linear in nature (Kugler and Turvey 1987). As an essentially physical phenomenon it should, therefore, be possible to describe walking in terms of the physics of oscillating systems. Walking is periodic in nature with the system in dynamic equilibrium and, as such, also fulfills the definition of an oscillator. Power for walking is supplied rhythmically with temporal consistency (Winter 1980). Modelling locomotion as a pendular activity has been used in human gait (McMahon 1984) and in quadrupedal gait (Kugler and Turvey 1987).

The force-driven harmonic oscillator (FDHO) is one model that may be appropriate for human gait. As in walking, the FDHO requires a periodic forcing function to maintain its oscillations as a result of the gravitational, damping and stiffness forces which will tend to diminish the oscillations. The relatively constant period and amplitude of walking are also found in the FDHO. If walking is the biological equivalent

of an FDHO, the importance of frequency becomes apparent. In an FDHO, there is a particular frequency, the resonant frequency, which requires the minimum force to maintain its oscillation. The minimization of force (i.e. muscle activity) in walking will result in a decreased need for oxygen. Hence frequency as a predictor of minimal energy may be an a posteriori fact of a biological system obeying the biophysical laws governing oscillators. The purpose of this study was to determine whether the resonant frequency of the force-driven harmonic oscillator predicts the freely chosen frequency adopted in walking.

Theory

The behavior of a force-driven harmonic oscillator is described by the following formula:

$$F = \frac{\theta L M}{\sqrt{\dfrac{1}{\left(\omega_0^2 - \omega^2\right)^2 + 4(\beta/2M)^2\omega^2}}},$$

where:

F = driving force (N),
θ = amplitude (degrees),
L = simple pendulum equivalent length (m),
M = mass (kg),
ω_0 = resonant frequency (hertz),
ω = actual frequency (hertz),
β = damping coefficient ($\mathrm{kg \cdot s^{-1}}$),
g = gravity ($9.81\ \mathrm{m \cdot s^{-2}}$),
and the resonant frequency is:

$$\omega_0 = \frac{1}{2\pi\sqrt{\dfrac{L}{g}}}$$

(Kugler et al. 1980).

The driving force required to drive this oscillator is a minimum whenever the term:

$$\left(\omega_0^2 - \omega^2\right) = 0,$$

for real numbers F is a minimum when:

$$\omega_0 - \omega = 0,$$

or

$$\omega_0 = \omega.$$

Therefore, if humans are sensitive to the minimal force required to drive their limbs, and if those limbs behave as force-driven harmonic oscillators, the actual frequency will be equal to the predicted frequency:

$$\omega_0 = \omega = \frac{1}{2\pi\sqrt{\dfrac{L}{g}}},$$

or in terms of the period:

$$\boxed{\tau = 2\pi\sqrt{\frac{L}{g}}} \quad \text{predicted}^1.$$

Kugler and Turvey (1987) suggested an adjustment to the above formula applying an integer multiple of 2 to the gravitational term. This produced much better predictive capabilities of the model for quadrupeds. The second predictive formula that may be used, therefore, is:

$$\omega_0 = \omega = \frac{1}{2\pi\sqrt{\dfrac{L}{ng}}},$$

where n = constant (2) or in terms of the period:

$$\boxed{\tau = 2\pi\sqrt{\frac{L}{2g}}} \quad \text{predicted}^2.$$

The value for L (distance from axis of rotation to CM of pendulum system) in each case is calculated using standard anthropometric data and mathematical calculations. The thigh-shank-foot can be assumed to be a single rigid body attached at the hip joint by a frictionless pin joint. The mass of each segment can also be assumed to be a point located at the CM of each segment. The added mass is assumed to be a point mass located at the malleolus.

First, the locations of the CM of each segment can be calculated using the data provided in Dempster (1955). The location of the CM of the lower extremity-added mass system can be calculated using the following formula:

$$h_{sys} = \frac{\sum_{i=1}^{4} m_i h_i}{M},$$

where:
h_{sys} = distance from axis of rotation to CM of system,
m_i = masses for respective segments and added mass,
h_i = distance from CM of respective segments and added mass location to axis of rotation,
M = total mass of system.

Similarly, the moment of inertia of each segment can be given by:

$$I_{segment} = m_{seg} \times (\rho \times h)^2,$$

where:
m_{seg} = segment mass,
ρ = segment radius of gyration about segment CM (Winter 1980),
h = distance from segment CM to axis of pendulum.

The moment of inertia of the added mass can be assumed to be:

$$I_{AM} = \text{added mass} \times \text{leg length}^2,$$

where I_{AM} = added mass moment of inertia,

The system moment of inertia about the axis of rotation can then be calculated using the parallel axis theorem as:

$$I_{sys} = \sum_{i=1}^{3} \left(I_i + m_i D_i^2\right) + I_{AM},$$

where:
I_i = segment moment of inertia,
m_i = segment mass,
D_i = distance from segment CM to axis of rotation.

The value for L necessary for each pendulum as defined in Kugler and Turvey (1987), therefore, is given by:

$$L = \frac{I_{\text{system}}}{M_{\text{system}} \times D_{\text{system}}}.$$

Predicted periods can be determined by substituting the value of L (meters), π (3.141), and acceleration due to gravity (9.81 ms^{-2}) into the adjusted and non-adjusted formulae to calculate the period of the oscillation.

Methods

Subjects

Twenty-four young, healthy adults, 16 males and 8 females, served as subjects in this study. Participants were screened so that persons with known pathology of the trunk or lower extremity were excluded from the study. A visual inspection of each subject's gait revealed no noticeable abnormalities or pathological gait patterns. In addition, subjects were included whose heights range from very short (1.57 m) to very tall (1.91 m) with a mean of 1.76 m ±0.10. Prior to participation in the study, subjects were asked to read and sign an informed consent form in accordance with University policy. Individual subject characteristics are presented in table 1.

Protocol

On entering the testing area, any questions the subject might have regarding the study were answered at this time. The subject's height and weight were immediately recorded. Anthropometric measures of the lower extremity during stance were taken by an experienced Physical Therapist. These measures included: (1) thigh length (greater

Table 1
Individual subject characteristics.

Subject	Sex	Height	Weight	Leg	Thigh	Shank	Foot
1	F	1.57	456	0.69	0.37	0.33	0.12
2	M	1.74	804	0.79	0.39	0.40	0.14
3	M	1.83	765	0.84	0.43	0.41	0.15
4	F	1.68	594	0.79	0.43	0.41	0.13
5	F	1.66	520	0.78	0.38	0.38	0.13
6	F	1.71	647	0.77	0.42	0.35	0.14
7	M	1.84	814	0.85	0.42	0.43	0.16
8	F	1.60	515	0.73	0.37	0.36	0.13
9	M	1.83	736	0.92	0.48	0.44	0.16
10	M	1.89	755	0.90	0.45	0.45	0.15
11	M	1.72	765	0.86	0.47	0.39	0.14
12	F	1.80	657	0.87	0.45	0.42	0.14
13	M	1.80	774	0.84	0.43	0.41	0.14
14	M	1.91	878	0.90	0.46	0.44	0.16
15	M	1.79	707	0.85	0.44	0.41	0.13
16	M	1.67	795	0.75	0.38	0.37	0.13
17	M	1.81	750	0.83	0.46	0.37	0.14
18	M	1.74	682	0.81	0.41	0.40	0.14
19	M	1.75	706	0.81	0.41	0.40	0.14
20	F	1.61	559	0.77	0.38	0.37	0.13
21	M	1.90	893	0.91	0.49	0.42	0.15
22	F	1.69	652	0.79	0.43	0.36	0.13
23	M	1.89	750	0.86	0.45	0.41	0.14
24	M	1.72	697	0.79	0.43	0.36	0.14
Mean		1.76	702	0.82	0.43	0.40	0.14
S.D.		0.10	112	0.06	0.03	0.03	0.01

Note: Length: meters, Weight: Newtons

trochanter to the lateral condyle of the knee, (2) shank length (lateral condyle of the knee to lateral malleolus, (3) foot length (lateral malleolus to fifth metatarsal), and (4) leg length (greater trochanter to lateral malleolus). Measurements of both limbs were taken and subjects with leg lengths which differed by more than 1.5 cm were excluded from the main study.

The subjects then completed walks under each of the following conditions: no added mass (0 kg), and with added masses at the ankle of 2.27, 4.55 and 6.82 kg on each leg. Thus each subject completed a total of five walks in each of the four mass conditions. A walk consisted of ambulation at the subject's preferred rate around the perimeter of a large gymnasium-sized testing area. On each straight stretch of a walk, the experimenter began count of the cycles of the lower right extremity. A cycle was considered from the heel strike of the foot of one limb to the next heel strike of the same limb. Concurrent with the commencement of cycle count, the experimenter started the stop-watch. On the completion of 20 cycles, the experimenter stopped the stop-watch, noted the time, and prepared the subject for the next walk. Thus five estimates of cycle counts were made for each subject/condition. The ordering of each condition was determined beforehand and assigned to the subject on entering the gymnasium. To avoid ordering bias, one of the 24 possible orders of conditions (combinations of load) was randomly assigned to the subject on entering the gymnasium.

Statistical procedures

The three conditions; actual, predicted^1, and predicted^2; were compared using a repeated measures analysis of variance. The same procedure was also used to compare the added mass levels. A linear best fit was then calculated across the load levels for each of the calculated periods and the slopes of the three lines were compared using a procedure outlined in Neter et al. (1985).

Results

The individual subject actual and predicted stride periods for all added mass levels is presented in table 2. In all cases, in both the actual and the two predicted cases, the stride period increased as a function of the added mass. The increase in each of the conditions was statistically significant ($p < 0.05$). Significant differences were observed between the actual stride period and the predicted^1 period at each added mass level ($p < 0.05$). This difference ranged from 28.8% to 37.3% over the added mass levels with the predicted^1 period always being greater than the actual period. However, the difference between the actual period and the predicted^2 period was not statistically significant. In this case, the percent differences ranged from 0.3% to 5.1%. In addition, the differences between two predicted stride periods was statistically significant at all mass levels.

because it changed the leg cm and thus L

Table 2
Individual subject actual and predicted periods for all conditions.

Subject	0 kg			2.27 kg			4.55 kg			6.82 kg		
	Act.	Pred.^{1}	Pred.^{2}	Act.	Pred.^{1}	Pred.^{2}	Act.	Pred.^{1}	Pred.^{2}	Act.	Pred.^{1}	Pred.^{2}
1	1.087	1.402	0.992	1.181	1.517	1.072	1.247	1.563	1.105	1.319	1.588	1.123
2	1.091	1.490	1.054	1.114	1.587	1.116	1.152	1.626	1.150	1.248	1.655	1.171
3	1.106	1.540	1.089	1.252	1.632	1.154	1.477	1.681	1.188	1.515	1.711	1.210
4	1.070	1.537	1.087	1.109	1.647	1.165	1.168	1.698	1.201	1.342	1.728	1.222
5	1.078	1.458	1.031	1.120	1.571	1.111	1.220	1.620	1.146	1.324	1.647	1.165
6	1.017	1.478	1.045	1.093	1.572	1.111	1.121	1.618	1.144	1.222	1.645	1.163
7	1.103	1.544	1.092	1.131	1.633	1.155	1.172	1.682	1.189	1.223	1.712	1.211
8	1.024	1.435	1.094	1.038	1.547	1.094	1.081	1.594	1.127	1.127	1.621	1.146
9	1.167	1.610	1.138	1.239	1.707	1.207	1.350	1.757	1.243	1.440	1.788	1.265
10	1.173	1.584	1.120	1.238	1.683	1.190	1.243	1.734	1.226	1.273	1.766	1.249
11	1.037	1.565	1.107	1.076	1.654	1.169	1.099	1.702	1.203	1.134	1.731	1.224
12	1.030	1.566	1.107	1.102	1.670	1.181	1.127	1.721	1.217	1.225	1.751	1.238
13	1.155	1.539	1.088	1.253	1.631	1.153	1.352	1.680	1.188	1.445	1.710	1.210
14	1.096	1.591	1.125	1.129	1.677	1.186	1.213	1.725	1.220	1.291	1.756	1.242
15	1.152	1.547	1.094	1.253	1.646	1.253	1.378	1.696	1.199	1.419	1.726	1.221
16	1.019	1.453	1.027	1.027	1.539	1.088	1.048	1.585	1.121	1.123	1.613	1.141
17	1.096	1.541	1.098	1.138	1.628	1.151	1.161	1.674	1.183	1.206	1.703	1.204
18	1.117	1.510	1.068	1.179	1.609	1.138	1.223	1.658	1.172	1.368	1.688	1.193
19	1.044	1.511	1.068	1.083	1.607	1.136	1.210	1.656	1.171	1.303	1.192	1.685
20	1.095	1.455	1.029	1.167	1.562	1.104	1.329	1.610	1.139	1.145	1.638	1.158
21	1.107	1.608	1.137	1.178	1.691	1.196	1.219	1.738	1.229	1.288	1.769	1.251
22	1.088	1.500	1.060	1.154	1.595	1.128	1.189	1.642	1.161	1.271	1.670	1.181
23	0.978	1.558	1.102	1.067	1.652	1.168	1.121	1.701	1.203	1.157	1.732	1.224
24	1.085	1.498	1.059	1.128	1.586	1.123	1.218	1.634	1.155	1.362	1.662	1.175
Mean	1.084	1.522	1.081	1.144	1.618	1.151	1.213	1.666	1.178	1.282	1.675	1.220
S.D.	0.049	0.055	0.037	0.067	0.051	0.043	0.103	0.052	0.037	0.108	0.116	0.106

Fig. 1 is a graphical representation of the lines of best fit for each of the three conditions. The test for the difference in the slopes of these three equations resulted in no statistically significant difference ($p <$ 0.05).

Discussion

The present experiment was designed to determine whether the frequency at which subjects ambulated at a preferred rate was predictable as the minimal amount of force required to drive a force-driven harmonic oscillator. Results from these data supported the hypothesis

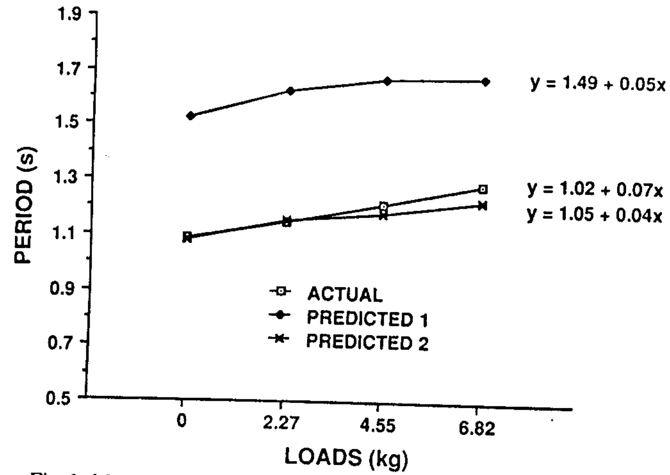

Fig. 1. Mean predicted and actual frequencies under all conditions.

that the resonant frequency of a harmonic oscillator can accurately predict that chosen by the subjects when appropriate adjustments are made to the formula based on an optimization criterion of minimum force.

However, the predicted[1] or resonant frequency of the oscillator consistently underestimated the actual frequency at all of the added mass levels. Why should the FDHO underpredict the frequency actually chosen by an individual? The assumption of an idealized oscillator – that the lower limb is considered a rigid lever in this model – may have served to produce the consistent difference between predicted[1] and actual frequencies. The effect of a multiple joint system on the momentum gained by the limb, is to decrease the swing phase, and hence the period (Northrip et al. 1974). Thus, another appropriate (and complex) model might combine the physical properties of biological oscillators with the biomechanical properties of the limbs

A second explanation of the underprediction of the model may be related to the notion of optimization. Theoretical phase portraits of energy cost versus frequency (Kugler et al. 1980) reveal that at low masses, frequency may not be a crucial variable (fig. 2). As the added mass increases, the importance of the appropriate frequency to energy cost gains status. Thus, when normal, healthy, non-fatigued individuals ambulate, the mass of the lower limbs may be inconsequential relative

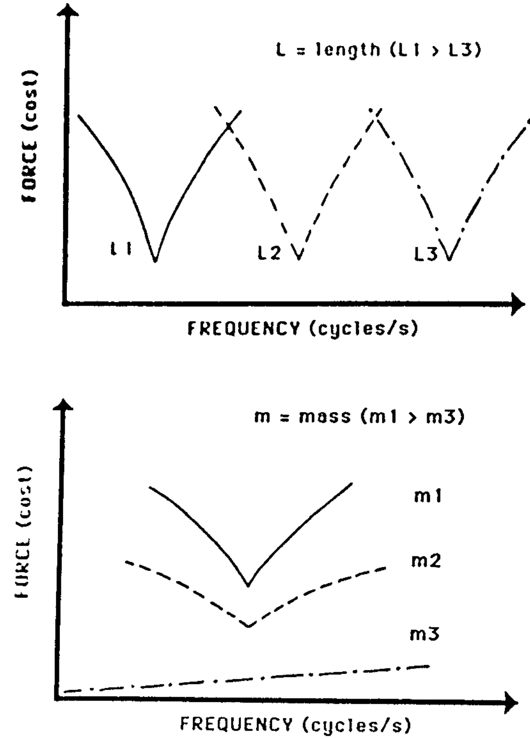

Fig. 2. Theoretical phase portraits of a force-driven harmonic oscillator (after Kugler et al. 1980).

to the supporting musculo-skeletal and biomechanical system. Intuitively, this will be so until the added mass on the system is increased to a point where energy and, according to the model, frequency become crucial variables.

Kugler and Turvey (1987) reported that, in quadrupedal gait, if the gravitational constant was multiplied by a factor of 2, the preferred frequency could be predicted accurately by the natural frequency of the FDHO. By iterating on the predicted frequency with constants ranging from 1.95 to 2.05, it was found that, in this study, a value of 2.0 for n gave a prediction that matched the self-selected frequency.

When an integer multiple of 2 was applied to the gravitational constant (predicted[2] condition) as suggested by Kugler and Turvey (1987), the model became an accurate predictor of the freely chosen frequency. The predicted[2] model was particularly accurate at the 0 kg and 2.27 kg added mass levels (0.3% and 0.6% difference from the actual frequency respectively) but slightly overestimated the frequency at the 4.55 kg and 6.82 kg added mass levels (3.0% and 5.1% difference respectively). The suggestion of these results is that the walking of bipeds is organized around gravity multiples of 2.

Several factors may account for the model not being as effective at the greater mass levels. One factor relates to the uniformity of the relationship between the mass and pendulum length which normally scales as mass proportional to the length cubed. Turvey et al. (1988) have shown that if the relation of mass to length of a set of virtual single wrist pendulum systems is not constant, inhomogeneities are reflected in the relationship of period to mass and length. In this study the added mass would serve to disrupt the system. As more mass was added the inhomogeneities would have increased resulting in greater discrepencies between the preferred and predicted frequencies. A second consideration is the effect of an increased moment of inertia at the higher loads. An inertia term as described in the predictive formulation of Turvey et al. (1988) may perhaps provide a better account of walking under loaded conditions. Finally, the wearing of weights on one's ankle is neither a 'normal' ecological event nor is it a well learned skill. A neuromuscular reorganization may be necessary to provide the appropriate muscle force for the new system to counteract changes in the moment of inertia. If indeed self-optimization drives the system it may take the actor some time to 'learn' and adopt the new optimum.

While references concerning metabolic costs form the basis for this study, no direct measures of O_2 consumption were made. Work is currently underway which not only measures metabolic costs, but also measures the biomechanical cost, in terms of mechanical energy, at self-selected and predicted frequencies. In summary, the findings of this research support the notion that motor control parameters emerge from the physical attributes of the system, and are not due to some a priori program prescription.

Conclusions

These findings lend support to a growing body of knowledge which points to the physical underpinnings of preferred modes of coordinated rhythmical movements. In particular, the periods of activities requiring

the raising and lowering of masses against gravity are predictable as the resonant period of the harmonic oscillator or some hybrid model (e.g. Turvey et al. 1988). The unique contribution of this study is the demonstration that the same allometric law (relating body segment mass and length) which applies to quadruped walking (Kugler and Turvey 1987) also applies to biped human walking. In addition, the use of the FDHO model has allowed the application of a simple pendulum equivalent rather than the necessity for a compound or double pendulum to describe walking.

It is likely that the underlying 'goal' of locomoting systems is self-optimization, in the form of minimal muscular requirements, or minimal metabolic cost and that resonance is the method by which it is achieved.

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