

A behavioral dynamics approach to modeling realistic pedestrian behavior

Stephane Bonneaud and William H. Warren

Abstract Realistic models of locomotion, accounting for both individual pedestrian behavior and crowd dynamics, are crucial for crowd simulation. Most existing pedestrian models have been based on ad-hoc rules of interaction and parameters, or on theoretical frameworks like physics-inspired approaches that are not cognitively grounded. Based on the cognitively-plausible behavioral dynamics approach, we argue here for a bottom-up approach, in which the local control laws for locomotor behavior are derived experimentally and the global crowd behavior is emergent. The behavioral dynamics approach describes human behavior in terms of stable, yet flexible behavioral patterns. It enabled us to build an empirically-grounded model of human locomotion that accounts for elementary locomotor behaviors. Based on our existing components, we then elaborate the model with two new components for wall avoidance and speed control for collision avoidance. We show how the model behaves with many stationary obstacles and interacting agents, and how it can be used in agent-based simulations. Five scenarios show how complex individual behavioral patterns and crowd dynamics patterns can emerge from the combination of our simple behavioral strategies. We argue that our model is parsimonious and simple, yet accounts realistically for individual locomotor behaviors while yielding plausible crowd dynamics, like lane formation. Our model and the behavioral dynamics approach thus provide a relevant framework for crowd simulation.

Key words: Empirically-grounded model, pedestrian behavior, behavioral dynamics, individual based model.

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1 Introduction

Crowd simulation would greatly benefit from models accounting reliably for both individual pedestrian behavior and crowd dynamics. The ability to accurately reproduce human locomotor dynamics and paths of travel, as well as self-organized patterns of crowd dynamics is necessary for urban planning, disaster management, or any research and application requiring artificial pedestrians to mimic human behavior [26]. As has recently been emphasized [18, 22, 26], realistic crowd simulation calls for stronger theoretical and empirical grounding in principles inspired by cognitive science research. Yet, as of today, building a realistic model, demonstrating that a model is realistic, or quantifying the extent to which it is realistic, are unsolved problems. When modeling crowd dynamics, one may measure and try to reproduce global crowd parameters, and yet the resulting individual trajectories can be highly unrealistic. We argue for a converse bottom-up approach, in which the local control laws for locomotor behavior are derived experimentally and the global crowd behavior is emergent, and also evaluated empirically. Such a generative model will not only produce accurate trajectories at the individual level, but is also likely to yield realistic collective behavior.

Thus, our focus here is on modeling human locomotor behavior, and investigating the properties of crowd behavior that emerges from local interactions. The first challenge to modeling locomotion is understanding the dynamics of an individual interacting with its environment to move toward goals and avoid obstacles. Hence, the modeling needs to be grounded in observations of human locomotion and based on theoretical principles governing human perception and action [27]. The second challenge is understanding what makes a model realistic, when it is impossible to validate its predictions in all possible crowd scenarios of locomotion. Hence, the model not only needs to behave realistically in particular scenarios, but it needs to capture the general behavioral dynamics of human locomotion [28]. Arguably, a prominent feature of human locomotion is our capacity to effortlessly locomote in highly unpredictable environments with stable, yet flexible behavioral patterns. In consequence, we believe that the theoretical framework of the model has to describe human behavior in broader terms than those of locomotion only, and should provide cognitively plausible conceptual tools for modeling stable, yet flexible behavior.

Most existing pedestrian models have been based on ad-hoc rules of interaction and parameters [18, 24], or on theoretical frameworks like physics-inspired approaches that are not cognitively grounded [13]. Recently, articles introducing approaches inspired by cognitive science and psychology have led the way to more plausible simulations [21, 23]. [21] advocate in particular for a synthetic modeling of the pedestrian perception system providing information to cognitively inspired heuristics controlling the movement. Their Gibsonian description of the agent's perception system [11], which others [23] have also advocated, explicitly base visual control on the information provided by the optic flow [29]. [23] went further in linking their model to the theory and data by arguing for a dynamical approach to modeling locomotion and by using behavioral strategies found in the cognitive science literature, namely the constant bearing strategy for avoiding obstacles [28, 5]

and the tau-dot strategy for decelerating to avoid collisions [30, 19]. In doing so, these models have been able to simulate a rich range of behavioral patterns, from the individual level of the pedestrian to patterns of collective behavior in crowds. Yet, while providing more plausible models, these approaches haven't shown why and to what extent they produce realistic patterns of crowd dynamics based on empirical observations. Furthermore, their assumptions are not always consistent with human data, e.g. the path of least energy hypothesis [21] may not fully account for preferred human trajectories [9], and the use of the constant bearing strategy [23] rather than a heading error strategy [7] to steer toward a stationary goal leads to unrealistic spiral trajectories.

By experimentally deriving the local control laws for locomotor behavior, we built a model that generates reliable individual trajectories in the general locomotor scenarios of moving towards a goal, while avoiding stationary and moving obstacles. The model is motivated by the cognitively-plausible behavioral dynamics approach [27], which synthesizes the ecological approach to perception [12] and the dynamical approach to action [17, 15]. The behavioral dynamics approach aims to provide a theoretical framework that accounts for the stable and flexible nature of human behavior. Applied to locomotion, it translates into a parsimonious model accounting for general human locomotor dynamics [7, 8, 28]. In this presentation, we first briefly describe the behavioral dynamics approach. Then, we describe our locomotion model and the control laws for four elementary locomotor behaviors. Finally, we show how the model can be used in agent-based simulations of crowd scenarios, including *many* stationary obstacles and *many* interacting agents. We discuss the performances of our model in terms of behavioral patterns in the context of computer simulation and animation.

2 A Behavioral Dynamics Approach to Locomotion

The behavioral dynamics approach [27] is an ecological, emergent, and distributed approach to behavior. Based on the ecological approach to perception and action [12], the individual is seen as coupled to its environment through information and control variables, such that action is governed by behavioral strategies or control laws. The locomotor trajectory is not prescribed by an internal planning process, but emerges from the interactions of the individual agent with its environment. The temporal evolution of behavior is thus not determined by either the agent or the environment alone, but control is distributed across both, through the conjunction of the agent-environment state, given the task constraints and the effective control laws. As such, control lies in the agent-environment system [12] and is distributed and self-organized.

The coupling between the agent and the environment is both mechanical, through forces exerted by the agent, and informational, through information fields in the environment that specify the state of affairs to the agent. From that coupling emerges a pattern of behavior, with a dynamics characterized by stable states, bifurcations,

hysteresis, attractors and repellers. As such, the approach characterizes the agent-environment interaction as a dynamical system, and the unfolding behavior as a trajectory in the state space of the system. It is this emergent behavior that is called the behavioral dynamics [27].

Locomotor behavior is thus treated as a particular case of agent-environment coupling. Understanding locomotion dynamics in this framework means explicating (1) the visual information used by the agent and (2) the control laws that use this information to modulate the agent's action so as to reach a goal while avoiding obstacles. To derive and test hypotheses about the informational variables and control laws, we depend on controlled experiments with human participants in real and virtual worlds [7, 8]. The results yield control laws of locomotion, which can then be implemented in model simulations of human behavior.

3 A Pedestrian Model

3.1 Empirically-Grounded Components

Both the theory and the empirical observations enabled us to describe a parsimonious model using behavioral rules under different environmental conditions that humans use when locomoting in an environment. We propose to decompose locomotion into a set of elementary behaviors that can be modeled individually. As a first approximation, these include (a) steering to a stationary goal, (b) avoiding a stationary obstacle, (c) intercepting a moving target, and (d) avoiding a moving obstacle. Subsequently, we hypothesize components for (e) wall following and (f) speed control. Our strategy [7] is to model each elementary behavior as a nonlinear dynamical system and then attempt to predict human behavior in more complex environments by linearly combining these components. We determined the behavioral variables as the heading (ϕ) or direction of travel of the agent (with respect to an allocentric reference axis), and the current turning rate ($\dot{\phi}$), assuming for the moment a constant speed of travel v .

The general idea of the model is to consider the directions of goals and obstacles as attractors and repellers, respectively, for the agent's heading. The simplest description of steering toward a stationary goal is for the agent to align its heading ϕ with the direction of the goal ψ_g , that is to bring the target-heading angle to zero ($\phi - \psi_g = 0$) as it moves forward, which defines an attractor in state space at $[\phi, \dot{\phi}] = [\psi_g, 0]$. The law controlling the angular acceleration of the agent's heading is then:

$$\ddot{\phi} = -b\dot{\phi} - k_g(\phi - \psi_g)(e^{-c_1 d_g} + c_2) \quad (1)$$

First, the equation is a second order dynamical system with a damping term $-b\dot{\phi}$, because a first-order system generates a turning rate that is unrealistic and unconstrained. Second, the backbone of the behavioral strategy is found in the term $k_g(\phi - \psi_g)$ which serves to null the heading error. Finally, a distance term

($e^{-c_1 d_g} + c_2$) exponentially decreases the goal's attraction with distance d_g ; the (positive) c_2 constant keeps it greater than zero, making the goal always attractive whatever the distance.

Conversely, for a stationary obstacle that lies in a bearing direction (ψ_o) at a distance d_o , the simplest description of obstacle avoidance is to magnify the obstacle-heading angle ($\phi - \psi_o > 0$), which defines a repeller at $[\phi, \dot{\phi}] = [\psi_o, 0]$. The dynamics of angular acceleration induced by the obstacle is given by:

$$\ddot{\phi}_o = k_o(\phi - \psi_o)(e^{-c_3|\phi - \psi_o|})(e^{-c_4 d_o}) \quad (2)$$

where $k_o(\phi - \psi_o)(e^{-c_3|\phi - \psi_o|})$ is the underlying law, specifying an exponential decay of the repulsion as the agent turns away from the obstacle and the obstacle-heading angle increases, so the agent bypasses the obstacle rather than spinning around. ($e^{-c_4 d_o}$) is the distance term. If the agent encounters an obstacle en route to a goal, these two components are superposed, such that the agent turns away from the obstacle locally while simultaneously attempting to null the heading error with the goal. The agent's path thus emerges from the evolving competition between attractors and repellers as it moves through the environment.

Steering toward a moving goal turns out to yield a different solution from the stationary case. Rather than turning toward the target, pedestrians try to keep the target in a constant bearing direction [5, 27, 20, 3] as it moves forward. Sailors have long used this strategy: if another ship remains at a constant compass bearing, one is on a collision course. The law controlling the agent's heading is thus to null change in the bearing direction of the moving target (ψ_m):

$$\ddot{\phi} = -b\dot{\phi} + k_m\dot{\psi}_m(d_m + c_1) \quad (3)$$

($d_m + c_1$) is a simple linear distance term, which compensates for the decrease in angular velocity with target distance; c_1 prevents the target's influence from dropping to zero as distance decreases.

Conversely, for a moving obstacle, the agent follows the inverse strategy by avoiding a constant bearing with the obstacle.

$$\ddot{\phi}_{mo} = k_{mo}(-\dot{\psi}_{mo})(e^{-c_5|\dot{\psi}_{mo}|})(e^{-c_6 d_{mo}}) \quad (4)$$

The term $k_{mo}(-\dot{\psi}_{mo})(e^{-c_5|\dot{\psi}_{mo}|})$ constitutes the core of the strategy, which serves to repel the agent's heading from the collision point, again with an exponential decay in repulsion as the agent turns away. The distance term ($e^{-c_6 d_{mo}}$) exponentially reduces the repulsion with distance from the target. In addition, we implemented this component so the repulsion would drop to zero as the agent passed the obstacle, when the obstacle-bearing angle was greater than $\pi/2$: $\left(1 - \frac{\text{sgn}(|\phi - \psi_{mo}| - \frac{\pi}{2}) + 1}{2}\right)$.

3.2 Wall avoidance

We have considered several models to account for how humans interact with walls, e.g. walls are treated as a set of point-obstacles whose repulsion is a weighted distribution along the barrier length [10], or as a solid obstacle with a repulsion term based on the barrier’s visual angle [14]. Our model accounts for obstacle avoidance in general terms, and we propose to have agent interact with walls using an analogous control law. As shown in figure 1, we make the assumption that an individual interacts with a “virtual” point-obstacle, that corresponds to the wall’s closest point. This point is treated as a repeller for the agent’s heading following the stationary obstacle component described previously. A similar approach was used in other models, including Helbing’s original model [13]. We are in the process of testing it against human data.

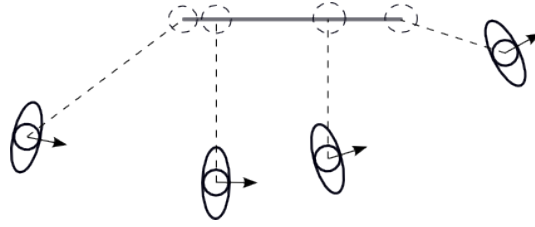


Fig. 1: An individual considers a barrier as a punctual obstacle which position depends on its relative position to the barrier. The figure shows different points on the barrier with which the individual is interacting depending on its own position.

3.3 Tau-dot based speed control

Lee [19] showed that the visual system might control behavior based on the temporal proximity of objects, rather than their distance per se. He showed that the time-to-collision with an object is specified by its relative rate of optical expansion, the optical variable “tau”, under certain conditions. Further, the rate of change in this variable, “tau-dot”, can be used to prevent collision by decelerating to hold tau-dot constant at a value of -0.5 [19]. As shown in [30, 16], the tau-dot strategy seems to be used by humans to control deceleration during braking, even when spatial information is available (but see [6] for a related strategy).

We propose here to use a strategy equivalent to the tau-dot strategy to provide agents with a speed control and collision avoidance component. For purpose of simulation, we compute the equivalent value of deceleration from physical variables [30] (Appendix A12), rather than using the tau-dot variable itself as input. This differs in its formulation and implementation from [23]. Let’s consider an agent

walking with an object in its field of view. The general idea is for the agent to adopt a preferred speed v when there is no risk of imminent collision, and to decelerate to avoid collision based on τ when the object is in its path. We therefore express the dynamics on $\dot{v}$ as follows:

$$\dot{v} = s_g (v_{pref} - v) - s_o \frac{v^2}{2z} e^{(-c_5|\phi-\psi|)} \quad (5)$$

With z the distance to the perceived object, v the agent's velocity, ϕ the agent's heading, and ψ the bearing angle with the perceived object. s_g , s_o , and c_5 are constants.

The first term, $s_g(v_{pref} - v)$, describes an attractor for the agent's speed in state space at $[v, \dot{v}] = [v_{pref}, 0]$. The second term computes the required deceleration $s_o \frac{v^2}{2z}$. The term $e^{(-c_5|\phi-\psi|)}$ exponentially increases as the obstacle-heading angle $(\phi - \psi)$ goes to zero. Its formulation comes from our moving obstacle component, making the agent slow down only if the risk of collision is located in front of it.

4 Simulations

[7, 8, 28] extensively review and detail how the model accounts for observations of human behavior. Therefore, we propose to show how adding our wall avoidance and tau-dot-like components produce credible simulations of pedestrian locomotion. And, we study five complex scenarios of crowd behavior. Three scenarios focus on interactions between pedestrians walking in corridors. The last two focus on a pedestrian finding its way on a crowded plaza with (1) N stationary obstacles, and then (2) M agents walking in various directions.

4.1 Elementary scenarios of interaction

(Scenario S1) Travel in a corridor

This scenario sets an agent at one end of a corridor with its goal outside of the corridor at the opposite end. The corridor is made up of six short walls, making up a winding corridor structure. The agent is initially located at the entrance of the corridor, and its goal is positioned so that the agent, following the walls of the corridor, in some places has to walk away from the goal's position. Figure 2a shows the resulting spatial trajectory of the agent, which is centered in the first segment of the corridor and then follows one wall until it reaches the corridor's end and turns towards its goal. Figure 2c represents the temporal evolution of wall repulsion and goal attraction: the time series of the repulsion components for each of the six walls are in grayscale, and that for the attraction of the goal is in red. Walls 4 and 5, which are located between the agent and the goal, provide the dominant repulsion components.

(Scenario S2) Two pedestrians walking towards each other in a corridor

Two agents are set at each end of a linear corridor (3 meters wide and 10 meters long) with their respective goals at the opposite end. The agents' initial positions are centered in the corridor, but noise is introduced to slightly randomize their initial locations and initial headings. Figure 2b shows sample spatial trajectories. We can see that the agents avoid each other while also remaining within the corridor's walls. Figure 2d takes the perspective of the "red" agent in figure 2b, and shows the time series of each of its obstacle avoidance components. We can see how its repulsion from the other agent evolves in time, and how its behavior is influenced. As the "red" agent tries to avoid the other agent, the repulsion of the walls increases.

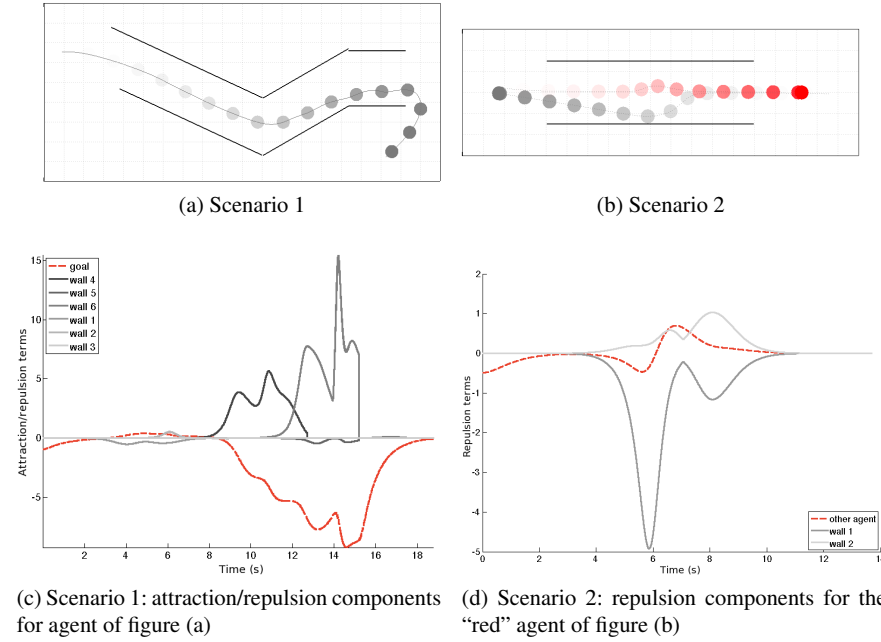


Fig. 2: Scenarios S1 (left) and S2 (right). (a) and (b) Agent spatial trajectories for S1 and S2: agent locations are visualized with colored disks, which value is function of time: most recent locations have the highest color value. (c) and (d) Time series of the model's components (attraction and repulsion terms) for S1 and S2. On the left, temporal evolution of wall repulsion (grey plots) and goal attraction (red plot). On the right, temporal evolution of the wall repulsion (grey plots) and other agent repulsion (red plot).

4.2 Complex scenarios of interaction

(Scenario S3) *One pedestrian passing through a crowd in a corridor*

This scenario is comparable to scenario 2 except that a group of nine agents is at one end of the corridor. Figure 3a shows the evolution in time of the simulation. Results show how the agent on the left finds its way through the middle of the group, and how the others avoid it while remaining within the corridor. Figure 3b shows the speeds of each agents. The red plot shows the time series of the speed of the “red” agent in the top figures, and the orange plot is the time series of the mean speed of all agents. In this figure, gray level codes the position of agents in the group, with the darkest grays indicating the front row. At the beginning of the trial, all agents slow down very slightly when entering the corridor. The front row keeps a nearly constant speed, whereas the second and third rows slow down more to avoid the front row. As the front row moves ahead, the second row slowly increases its speed, followed by the third row. As the group meets the “red” agent coming from the opposite direction, the front row slows down and moves aside –particularly a2, avoiding a head-on collision– followed by the second and third rows as they avoid agents who have moved in front of them.

(Scenario S4) *Two crowds passing through in opposite directions*

The scenario puts two groups of nine agents each walking towards each other. Their positions and headings are initialized with a slight noise to generate more irregular group configurations. Figure 4 shows the evolution in time of the agents’ spatial trajectories. Note that agents naturally form lanes.

(Scenario S5) *N stationary obstacles*

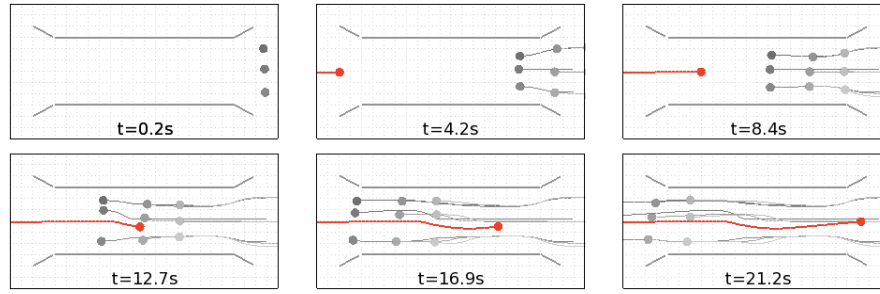
In this scenario, 30 stationary obstacles are randomly distributed in between the initial position of the agent and its goal. Figure 5a shows how the agent finds its way through the obstacles.

(Scenario S6) *M moving obstacles*

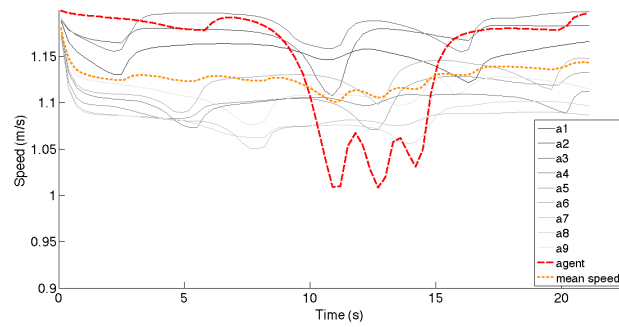
Finally, the last scenario shows how an agent finds its way in an hall where many other agents are walking. Four flows of agents are used as shown in figure 5b. Each flow is set on one of the side of the hall and generates agents that have to cross the hall to a goal on the other side. The figure shows how agents steer through the crowd toward their respective goals without collisions.

5 Discussion

We argue that our model is parsimonious, with few parameters. Each component is based on empirical data and built in the coherent theoretical framework of behavioral dynamics. Components have fixed parameters, and generalize to new scenar-



(a) Spatial trajectories



(b) Time series of speed

Fig. 3: Scenario S3. (a) Snapshots of different simulation time steps showing the spatial trajectories and positions of the different agents. (b) Speed time series, for the agents in the group in grey, and the “red” agent in red; the orange plot is the mean speed of all agents.

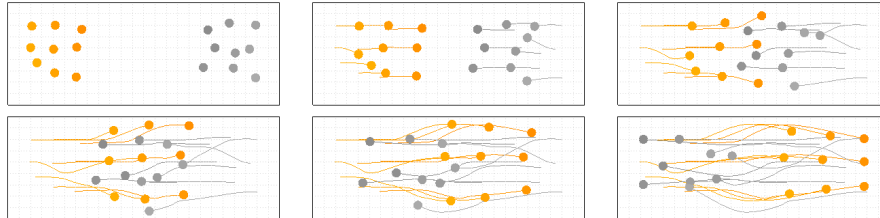


Fig. 4: Scenario S4. Snapshots of different simulation time steps showing the spatial trajectories and positions of the agents.

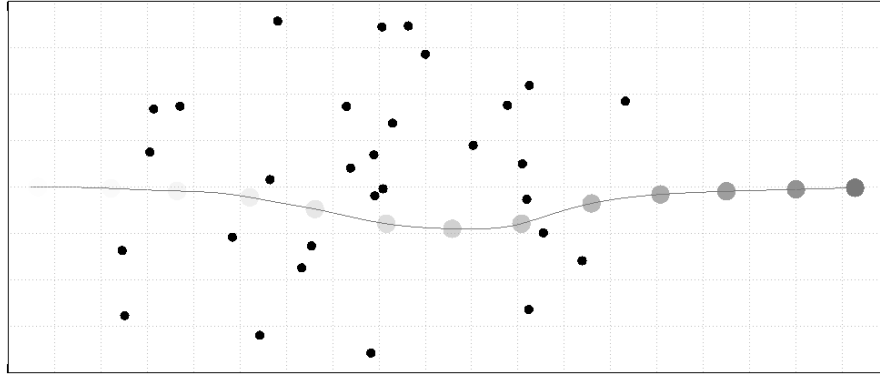
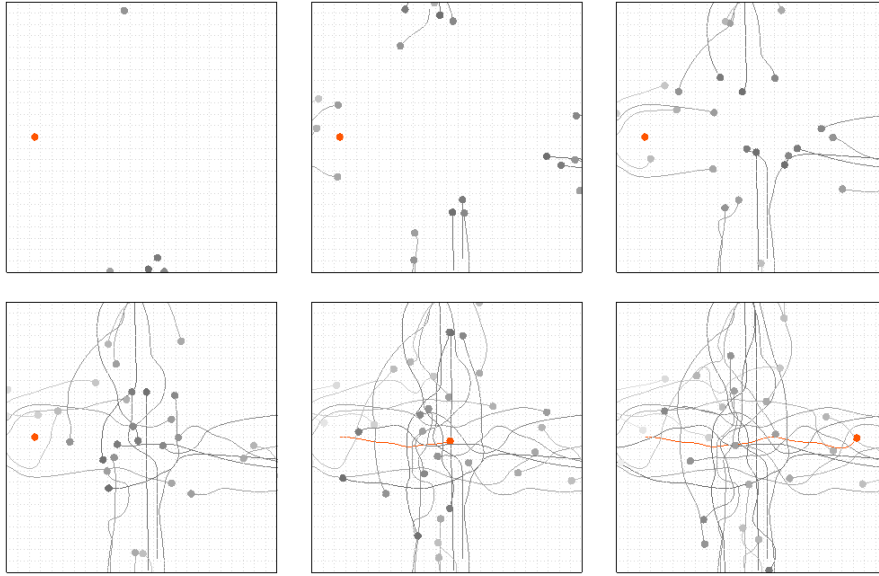
(a) N stationary obstacles(b) M agents

Fig. 5: Scenarios S5 (top) and S6 (bottom). (a) Agent spatial trajectories (low color value corresponds to older timesteps) with black disks corresponding to stationary obstacles. (b) Snapshots of different time steps showing agent spatial trajectories and positions.

ios. Only the parameter of the distance term c_6 of the moving obstacle component has been slightly modified to account for the dynamics of small groups, and we are currently investigating this collective behavior [1, 25, 2]. Both the theoretical approach and the components therefore provide a fruitful ground for finding realistic description of human behavior in complex scenarios of locomotion. We show that our model produces plausible individual trajectories in various scenarios from wall avoidance to crowd dynamics. The different scenarios show that agents follow walls without collision (S1), even when avoiding oncoming agents (S2). Individual agents exhibit smooth trajectories around obstacles with plausible paths to their goals (S5), even in highly dynamic environments (S6). Scaling up to groups of agents, the model still produces plausible trajectories. Groups smoothly split to avoid oncoming agents (S3), or intermingle without collisions, self-organizing into lanes (S4).

Yet, further studies of human behavior are required. The wall avoidance component is not based on observations of human behavior. And it is unknown how humans interact with walls in different situations: walls may be alternatively wide obstacles to walk around or long barriers to follow. The tau-dot-like strategy is based on human experiments, but our component is not fully linked to data, and the ratio of deceleration to turning still has to be adjusted to recent observations [4]. While humans avoid collisions by both decelerating and changing their heading, how they combine these two strategies is currently being modeled. The behavioral dynamics approach explains how locomotion emerges from the interactions of the agent with its environment, and, in this case, that the combination of speed and heading control is an online process from which emerges the locomotor path. And as the agent turns to avoid a collision, it might have to slow down less, or as it slows down, it might have to turn less. The model, by combining simple strategies, yields complex agent behavioral patterns. Yet, complex human behavioral patterns are still not described and quantified sufficiently to know if we fully account for them.

Arguing for the realism of a locomotion model for crowd simulation is still an unsolved problem. Our four elementary components account for individual locomotor paths in various scenarios. As such, these elementary components provide a platform for the design of new components. For obstacle avoidance for instance, our component has been built using observations of individuals interacting with poles, i.e. obstacles that can be assimilated to punctate objects. With more complex objects, e.g. a long table, it is likely that the stationary obstacle components have to be revised. This may have a cognitive reality, and if humans have simple behavioral strategies to locomote, they probably use them from within highly adaptive behaviors. Last, we show that our model scales up smoothly to scenarios of crowd dynamics, and reproduces in particular lane formation patterns. Yet, crowd dynamics in general requires further empirical study, e.g. we don't know how to fully characterize lane formation. Further experimental investigation of patterns of crowd dynamics is necessary, which we have started doing with small groups of pedestrians [1, 25, 2].

6 Conclusion

One of the main issues in crowd simulation is the lack of human data to constrain and validate pedestrian models. We described here a locomotion model grounded in observations of human behavior, that generates the same spatial trajectories as humans in four elementary tasks. We then elaborated our model with two new components to account for wall avoidance and speed control for collision avoidance. These two components were not built directly from observations of human walking as were the four others. For the wall avoidance component, we assumed our elementary stationary obstacle component to be a general formulation for avoiding the closest point on the obstacle. For the speed control component, we used a tau-dot-like strategy based on human data for the control of braking [30]. We then formulated these components within the behavioral dynamics framework, inspired by our previous model [28].

Finally, we showed how our model generates plausible trajectories and crowd patterns. Our two new components for wall avoidance and speed control for collision avoidance produce realistic results and scale up smoothly with several agents avoiding each other while remaining inside a corridor. Scenarios of crowd dynamics show that our model scales up when many agents are simulated. Smooth spatial trajectories of groups of agents intermingling without collisions show realistic patterns like lane formation. Yet, further observations and characterization of patterns of crowd dynamics are required. However, we believe that building an empirical model of individual locomotor behavior and using it, bottom-up, to account for emergent crowd behavior is a promising path to a realistic model of crowd dynamics. In this effort, a scientific theoretical framework accounting for the dynamics of human perception and action [27] offers a coherent modeling framework. By deriving the components from empirical data, we can maintain the parsimony of the model while accounting for human behavior.

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